DUE Wednesday, February 18, 2015.

Problems to work but not hand in:
§6.2: #1a, 3b, 6.
§6.3: #1, 5a, 7c.

Problems to turn in:
No WeBWork this week.
§6.2: #1d (2), 3c,d (4), 8∗ (3), 9 (2), 11 (3), 13† (5).

A. (5) Suppose \( h: \mathbb{R} \rightarrow \mathbb{R} \) is \( C^1 \), \( h(0) = 0 \), and \( |h'(x)| \leq c < 1 \) for all \( x \).
Define the mapping \( f: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) by
\[
f(u, v) = \begin{bmatrix} u + h(v) \\ v + h(u) \end{bmatrix}.
\]

(i) Prove that \( f \) has a \( C^1 \) local inverse function near any point \( \begin{bmatrix} u_0 \\ v_0 \end{bmatrix} \in \mathbb{R}^2 \).
(ii) Prove that \( f \) is one-to-one (i.e., if \( f(u, v) = f(s, t) \), then \( \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} s \\ t \end{bmatrix} \)). Hint: Apply the one-variable Mean Value Theorem to show \( u = s \) and \( v = t \).
(iii) Show that \( \|I - Df(u)\| \leq c \) for every \( u \in \mathbb{R}^2 \).
(iv) Prove that \( f \) is onto (i.e., given any \( y \in \mathbb{R}^2 \), there is some \( u \in \mathbb{R}^2 \) so that \( f(u) = y \)). Hint: Follow the set-up of the proof of the Inverse Function Theorem; the radius of the ball \( B(0, R) \) you pick should be related to \( \|y\| \).
(v) Deduce that \( f \) has a global \( C^1 \) inverse function \( g: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \).

§6.3: #4 (2), 7b (3).

B. (3) Prove that \( X = \{ x \in \mathbb{R}^4 : \|x\|^2 = 30 \) and \( 2x_1^3 - x_2^3 - 3x_3^3 = 0 \} \)
is a 2-dimensional manifold and give a basis for its tangent space at \( a = \begin{bmatrix} 4 \\ -1 \\ 2 \\ 3 \end{bmatrix} \).

Challenge problems (Turn in separately):
§6.2: #7 (4), 12 (4).

*Here and in #9 you must work with a general \( F \), not the ideal gas law.
†See the errata at [www.math.uga.edu/~shifrin/MultivariableErrors.pdf](http://www.math.uga.edu/~shifrin/MultivariableErrors.pdf) Start by doing the examples in part (b).