DUE Wednesday, October 8, 2014.

Problems to work but not hand in:
§3.3: #1, 2, 5, 16.
§3.4: #1a, 2b, 3, 4.

Problems to turn in:
WeBWork Homework 7
§3.2: #16* (3), 18 (3).

A. If the partial derivatives of $f$ are bounded on $U \subset \mathbb{R}^2$, prove that $f$ is continuous.

§3.3: #11† (3), 15 (3).

B. (3) Suppose $f: \mathbb{R}^2 \to \mathbb{R}$ is differentiable and we define $F\left(\begin{array} {c} u \\ v \end{array} \right) = f\left(\begin{array} {c} e^u \cos v \\ e^u \sin v \end{array} \right)$. Use the chain rule carefully to compute $(\frac{\partial F}{\partial u})^2 + (\frac{\partial F}{\partial v})^2$ in terms of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$, evaluated at the appropriate spot.

§3.4: #5‡ (3), 6 (2).

C. (3) Suppose $f: \mathbb{R}^2 \to \mathbb{R}$ is differentiable and $y^2 \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = 0$ everywhere. Find the level curves of $f$. (Hint: From knowing the direction of the gradient, you should figure out the slope of the level curve through \[ \begin{bmatrix} x \\ y \end{bmatrix} \] and solve the differential equation $dy/dx = \ldots$) Deduce that if we know $f$ on the $x$-axis, then we know $f$ everywhere in the plane: In particular, if $f\left(\begin{array} {c} x \\ 0 \end{array} \right) = F(x)$, give a formula for $f$.

Challenge problems (Turn in separately):
§3.2: #19 (5).

D. (4) Give a criterion (in terms of $\alpha, \beta, \gamma$, and $\delta$) for the function in §2.3, #17 to be differentiable at $0$.

E. Let $U \subset \mathbb{R}^2$ be a neighborhood of $0$ and $f: U \to \mathbb{R}$ is continuous. Suppose both partials $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exist at $0$ and only the former is continuous. Prove that $f$ is differentiable at $0$.

§3.4: #7 (3), 8 (3), 9 (3), 13 (4), 15 (4).

*Hint: Copy the (beginning of the) framework of the proof of Prop. 2.4.
†Please add the hypothesis that $f$ is differentiable. Also, contemplate to which WeBWork problem this result might be relevant.
‡Set this up explicitly as a chain rule problem, and apply the results of Example 3.