6.1 Volumes Using Cross-Sections

Exercises 6.1

Volumes by Slicing
Find the volumes of the solids in Exercises 1–10.

1. The solid lies between planes perpendicular to the x-axis at \( x = 0 \) and \( x = 4 \). The cross-sections perpendicular to the axis on the interval \( 0 \leq x \leq 4 \) are squares whose diagonals run from the parabola \( y = -\sqrt{x} \) to the parabola \( y = \sqrt{x} \).

2. The solid lies between planes perpendicular to the x-axis at \( x = -1 \) and \( x = 1 \). The cross-sections perpendicular to the x-axis are circular disks whose diameters run from the parabola \( y = x^2 \) to the parabola \( y = 2 - x^2 \).

3. The solid lies between planes perpendicular to the x-axis at \( x = -1 \) and \( x = 1 \). The cross-sections perpendicular to the x-axis between these planes are squares whose bases run from the semicircle \( y = -\sqrt{1 - x^2} \) to the semicircle \( y = \sqrt{1 - x^2} \).

4. The solid lies between planes perpendicular to the x-axis at \( x = -1 \) and \( x = 1 \). The cross-sections perpendicular to the x-axis between these planes are squares whose diagonals run from the semicircle \( y = -\sqrt{1 - x^2} \) to the semicircle \( y = \sqrt{1 - x^2} \).

5. The base of a solid is the region between the curve \( y = 2\sqrt{\sin x} \) and the interval \([0, \pi]\) on the x-axis. The cross-sections perpendicular to the x-axis are
   a. equilateral triangles with bases running from the x-axis to the curve as shown in the accompanying figure.
   b. squares with bases running from the x-axis to the curve.
   c. circles with diameter running from the x-axis to the curve \( y = \sec x \). The cross-sections perpendicular to the x-axis are
   a. circular disks with diameters running from the curve \( y = \tan x \) to the curve \( y = \sec x \).
   b. squares whose bases run from the curve \( y = \tan x \) to the curve \( y = \sec x \).

6. The base of a solid is the region bounded by the graphs of \( y = 3x \), \( y = 6 \), and \( x = 0 \). The cross-sections perpendicular to the x-axis are
   a. rectangles of height 10.
   b. rectangles of perimeter 20.

8. The base of a solid is the region bounded by the graphs of \( y = \sqrt{x} \) and \( y = x/2 \). The cross-sections perpendicular to the x-axis are
   a. isosceles triangles of height 6.
   b. semi-circles with diameters running across the base of the solid.

9. The solid lies between planes perpendicular to the y-axis at \( y = 0 \) and \( y = 2 \). The cross-sections perpendicular to the y-axis are circular disks with diameters running from the y-axis to the parabola \( x = \sqrt{5y^2} \).

10. The base of the solid is the disk \( x^2 + y^2 = 1 \). The cross-sections by planes perpendicular to the y-axis between \( y = -1 \) and \( y = 1 \) are isosceles right triangles with one leg in the disk.

11. Find the volume of the given tetrahedron. (Hint: Consider slices perpendicular to one of the labeled edges.)

12. Find the volume of the given pyramid, which has a square base of area 9 and height 5.

13. A twisted solid A square of side length \( s \) lies in a plane perpendicular to a line \( L \). One vertex of the square lies on \( L \). As this square moves a distance \( h \) along \( L \), the square turns one revolution about \( L \) to generate a corkscrew-like column with square cross-sections.
   a. Find the volume of the column.
   b. What will the volume be if the square turns twice instead of once? Give reasons for your answer.