Section 2.4C: Some Extra Practice Problems
Addendum to Lecture Notes

This has a few problems we didn’t get to in class. Outlines are provided to get you started. Figure out, though, how we decide which things to label and which formulas to use! Which variables do you ultimately solve for, and which get eliminated in the process of the solution?

Problems like these have been on homework and WebQuizzes. Give them a shot!

**Extra #1:** A piece of wire 27 inches long is cut into two pieces and each piece is bent into a square. If $x$ denotes the length (in inches) of one piece, express the total area $A$ enclosed by both squares (in sq.in.) in terms of $x$.

OUTLINE: If $x$ is one piece, let’s say $y$ is the other piece. Since the two pieces make up the whole wire, we get $x + y = 27$. Now, if you wrap $x$ inches of wire into a square, that means $x$ is its perimeter... can you figure out the side length and then the area?

VARIANTS: What if one piece is a square and one is an equilateral triangle? What if the pieces are circles?

**Extra #2:** A garden is in the shape of a rectangle. A sidewalk of width 6 ft surrounds the garden. One side of the garden has length 26 ft. The other side has length $x$. Express the area of the sidewalk as a function of $x$.

OUTLINE: The easiest thing to do here is to find the area of the entire region (garden and sidewalk) and subtract out the garden! First, we get the length by adding the garden’s length (26) plus two sidewalk widths (6 each), to get $w = 38$. Similarly, you can find the width is $x + 2(6)$, i.e. $x + 12$. Now, you can get the area of the whole shape, and you subtract the area of the garden.

ALTERNATIVE: It is possible to break up the garden into a bunch of individual rectangles and add them all. This approach is much slower, and it’s much easier to make a mistake.

**Extra #3:** A square is inscribed within another square by connecting the midpoints of the larger square. The edge length of the outer square is $x$. Express the area of the inner square as a function of $x$.

OUTLINE: The intention here is to figure out a bunch of side lengths for this problem. Let’s say the inner square has sides of length $s$. If that’s the case, its area is $s^2$. However, we need to get this in terms of $x$, so let’s figure out $s$ in terms of $x$.

Next, we notice that there’s a right triangle in each corner of the big square, and $s$ is its hypotenuse. That suggests that we should use the Pythagorean Theorem! To do this, you need to know the legs of those corner pieces.

Now, use the fact that the vertices of the inner square are *midpoints*. What does that tell you about the lengths of those legs? Once you figure that out, you can get $s$, and then you can get the area.

REMARK: There is a cheap shortcut for this problem, if you think about it very carefully. Try drawing lines to break the bigger square into equal quarters. When you do that, you’ll see that each quarter has a shaded and an unshaded part of exactly the same size! Thus, the inner square should be exactly half the area of the outer square. Does this match the answer you get the other way when you simplify?