Sections 7.1 and 7.2 (part 2): Laws of Sine and Cosine

Last class, we saw the Law of Sines (LOS) and Law of Cosines (LOC). Each of these laws helps you solve oblique triangles. Basically, LOC is good when you have an SAS scenario, but most of the time LOS is better. Both of these laws are based on the idea of knowing a complete “opposing pair”.

These are tough laws to apply properly, so we’re going to have a lot of practice today in different scenarios. In some of these scenarios, we’ll have multiple triangles in the same drawing! In that case, it helps to figure out common sides or angles. Also, if you have a right triangle, SOHCAHTOA and Pythagoras are much easier tools to use than LOS and LOC.

**Ex 1:** In this trapezoid, angles $ABC$ and $BAD$ are right angles. Find the lengths of sides $CD$ and $AC$.

![Diagram of a trapezoid with angles and side lengths]

**HINT:** This problem actually involves two triangles. One of them is $\triangle ACD$. The other is a right triangle you get by drawing the height coming down from $C$. The two triangles have $CD$ in common!

**Bearings (see also Section 5.7 of textbook for more info)**

Up to now, we’ve usually indicated angles in standard position, meaning the initial side of the angle is the positive $x$-axis. However, for many situations (such as maps and sailing), people instead describe angles via bearings. A bearing has three parts:

1. N or S: This tells you if your initial side is the North or South axis.
2. A number in degrees: Tells you how much to rotate (between 0° and 90°).
3. E or W: This indicates if you want to rotate towards East or West.

Remember that N or S always comes first! The vertical axis is more important for bearings (whereas for reference angles and triangles, the horizontal axis was more important).

**Sample:** N40°E means “rotate 40° from North towards East”. It’s in Quadrant I, 50° away from the horizontal, so its standard measure is 50°.

**Ex 2:** For each angle, convert the standard position readings to bearings and vice versa.

(a) S80°W
(b) 300° in standard position
(c) 20° north of west (this is NOT a bearing!)

When you have bearings, take some care to make a crude sketch in word problems. It can be good to make one version of the picture with a little “compass rose” at each point with bearings, but then you may make a second cleaned-up version without the roses. After that, figure out which sides and angles you have, and solve the important parts of the triangle!

Be on the lookout for:

- Complements (angles that add to 90°) or supplements (add to 180°)
- Needing to add or subtract bearings (or their complements) to get the angles in a triangle

**Ex 3:** Town B is 34 miles from town A, at a bearing of S25°W. Town C is 84 miles from town A at a bearing of S9°E. Compute the distance from town B to town C.

**NOTE:** When you draw the two bearings, you’ll see the two angles add together. Thus, there’s a 34° angle between the sides $AB$ and $AC$.

**Ex 4:** Town C is 8 miles due east of town D. Town E is 20 miles from town C at a bearing (from C) of N54°E. How far apart are Towns D and E?

**NOTE:** Be careful: C is further east than D, so make sure C is on the RIGHT in your drawing!

**NOTE 2:** How would this problem change if the bearing were N54°W instead? What about S54°E?

**Challenging Examples**

The next problems are much more difficult. You’ll probably have to draw more on the figures than you might have expected. Also, you may find yourself using extra skills, like looking for supplementary angles or incorporating speeds. For many of these, it pays to (a) outline your steps ahead of time, and (b) store values in your calculator as you go (the “STO” button can be a big help, or use the “Ans” key).
Ex 5: A straight road makes an angle of $15^\circ$ with the horizontal. When the angle of elevation of the sun is $57^\circ$, a vertical pole at the side of the road casts a shadow 200 feet long directly down the road, as shown on the right. Find the length of the pole.

HINT: At first, you can only find one angle and side in the oblique triangle made by the road and pole, but that’s not enough info to solve. (You need at least three total pieces of info.) The key is to ALSO draw a right triangle underneath the road and the pole! It won’t help you get any more sides, since you already know the length of the common side, BUT it will help you find another angle.

Ex 6: A ship leaves port at 1:00 P.M. and sails in the direction N$38^\circ$W at a rate of 24 mph. Another ship leaves port at 1:30 P.M. and sails in the direction N$52^\circ$E at a rate of 19 mph.

(a) Approximately how far apart are the ships at 3:00 P.M.?
GETTING STARTED: The ships both started at port (say $C$ is port if you want). Our time units are in hours here. The first ship sailed for 2 hours, so it sailed $24 \cdot 2 = 48$ miles away; mark a point $A$ at the correct bearing with side length 48. Do a similar thing for the second ship to get a point $B$.

(b) What’s the bearing from the first ship (the one that left at 1:00 P.M.) to the second at 3:00 P.M.?
HINT: You’ll need to use the angle at $A$ AND the bearing of N$38^\circ$W to get this bearing. (If you draw two compass roses at $C$ and at $A$, you may spot two alternate interior angles of $38^\circ$.)

Ex 7: As shown in the figure, a cable car carries passengers from a point $A$, which is 1.2 miles from a point $B$ at the base of a mountain, to $P$ at the top of the mountain. The angles of elevation of $P$ from $A$ and $B$ are $\alpha = 24^\circ$ and $\beta = 70^\circ$, respectively.

(a) Find the length of the lift $AP$.
HINT: Much like in Ex 5, we don’t have enough info given in $\triangle ABP$ right away to solve it. However, we can draw a right triangle next to our oblique triangle. That will give us another angle in $\triangle ABP$, giving us an ASA situation which can use LOS.

(b) Find the height of the mountain.
HINT: There’s more than one right triangle in your drawing if you look carefully!

NOTE: Part (b) is actually just about the same kind of problem as the “two elevation angles” problem with a boat from the last day of Section 5.7’s coverage! Try comparing the method used there with the method used here; which do you prefer? (However, the method used here gives us more of an opportunity to solve for other sides and angles.)