Math 3000 Quiz 6 10/25/12

Instructions: Write your name on your paper. This quiz is to be your own work and the academic honesty policy of the University of Georgia applies. Calculators, books and notes are not allowed.

1. Let A be an $m \times n$ matrix and let W be a subspace of $\mathbb{R}^m$. Show that $\{\mathbf{x} \in \mathbb{R}^n \mid A\mathbf{x} \in W\}$ is a subspace of $\mathbb{R}^n$.

Let $V = \{\vec{x} \in \mathbb{R}^n \mid A\vec{x} \in W\}$.

1) $\vec{0} \in V$ since $A\vec{0} = \vec{0} \in W$ (since W is a subspace)

2) If $c \in \mathbb{R}$, $\vec{v} \in V$ then we must check $c\vec{v} \in V$:

$A(c\vec{v}) = c(A\vec{v}) \in W$ since $A\vec{v} \in W$ and W is a subspace. So $c\vec{v} \in V$.

3) If $\vec{u}, \vec{v} \in V$ then we must check $\vec{u} + \vec{v} \in V$:

$A(\vec{u} + \vec{v}) = A\vec{u} + A\vec{v} \in W$ since $A\vec{u}, A\vec{v} \in W$ and W is a subspace.

Since 1) - 3) hold, V is a subspace of $\mathbb{R}^n$.