NAME:

Math 3000 Quiz 5 10/18/12

Instructions: Write your name on your paper. This quiz is to be your own work and the academic honesty policy of the University of Georgia applies. Calculators, books and notes are not allowed.

1. Find a left inverse of the matrix

$$\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \xrightarrow{E_1} \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 2 & 1 \end{bmatrix} \xrightarrow{E_2} \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 0 & -3 \end{bmatrix} \xrightarrow{E_3} \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \xrightarrow{E_4} \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \xrightarrow{E_5} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = EA$$

$$E = E_5 E_4 E_3 E_2 E_1 = \begin{bmatrix} 1 & & \\ & -1 & \\ & & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & \\ & -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ 0 & 1 & \\ -2 & & 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ -1 & 1 & \\ & & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \\ -1 & -3 & 0 \end{bmatrix}$$

A left inverse C is the first 2 rows of E: $C = \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \end{bmatrix}$

2. Suppose that A is an $n \times n$ matrix with columns $\mathbf{a}_1, \ldots, \mathbf{a}_n$ and suppose that the columns are orthogonal (that is, if $i \neq j$ then the vectors $\mathbf{a}_i$ and $\mathbf{a}_j$ are orthogonal). Show that A^TA is a diagonal matrix (that is, if $i \neq j$ then the ij-entry of A^TA is 0). (Hint: How can you describe the ij-entry of a product of two matrices?)

The ij-entry of A^TA is

(i^{th} row of A^T) $\cdot$ (j^{th} column of A) = (i^{th} column of A) $\cdot$ (j^{th} column of A)

$$= \vec{a}_i \cdot \vec{a}_j$$

and by hypothesis this is 0 if $i \neq j$.