Math 8200, Spring 2011: Problem Set 4. Due Tuesday, February 22

I. Hatcher 1.1: 12, 13, 16(a)-(c), 20

II. A *topological group* is a group G equipped with a topology having the property that the multiplication map

$$\begin{aligned} \mu\colon\ G \times G &\to G \\ (a, b) &\mapsto ab \end{aligned}$$

and the inversion map $a \mapsto a^{-1}$ are both continuous. Let G be a topological group. Where e is the identity in G, define maps $i\colon\ G \to G \times G$ and $j\colon\ G \to G \times G$ by $i(a) = (a, e)$ and $j(a) = (e, a)$. Prove that, whenever $\alpha, \beta \in \pi_1(G, e)$, we have

$$\mu_*((i_*\alpha)(j_*\beta)) = \alpha\beta = \beta\alpha.$$

(Hint: Prove first that in $\pi_1(G \times G, (e, e))$ we have the identity $(i_*\alpha)(j_*\beta) = (j_*\beta)(i_*\alpha)$.) Note that this proves that π_1 of a topological group is always abelian.