Fall, 2014

MATH 4600/6600
PROBLEM SET #11

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DUE Friday, November 14, 2014.

Problems to work but not hand in:

Self-Test Problems and Exercises, Chapter 6: #12, 13, 16.

To hand in: Chapter 6: #12, 15, 30, 32, 41.

A. (from a recent GRE exam) A circular coin with radius 1 is tossed randomly so that when it lands it touches a 5×5 solid square. Assuming the location of its center is uniformly distributed, with what probability does the coin land entirely inside the square?

B. Suppose the Athens Transit bus is scheduled to arrive at your corner at 8:10 AM, but its actual arrival time is a normal random variable with mean 8:10 AM and standard deviation 40 seconds. Suppose you try to arrive at the corner at 8:09 AM, but your arrival time is actually normally distributed with mean 8:09 AM and standard deviation 30 seconds.

(i) What percentage of the time do you arrive at the corner before the bus is scheduled to arrive?

(ii) What percentage of the time do you arrive at the corner before the bus does?

(iii) If you arrive at the stop at 8:09 AM and the bus still hasn't come at 8:12 AM, what is the probability that you've already missed it?

C. Suppose X is a normal random variable with $\mu = 0$ and $\sigma = 1$ and Y is a normal random variable with $\mu = 1$. Find the standard deviation of Y if

(i) $P(X > Y) = 1/3$

(ii) $P(Y > 2X - 1) = 3/4$

(Hint: Consider $Z = aX + bY$ for appropriate a, b.)

D. Suppose X and Y are independent normal random variables with mean 0 and standard deviation σ. Say (X, Y) represents the location of a dart on a (suitably large) dartboard centered at $(0, 0)$.

(i) Suppose the bulls-eye has radius 4 inches and half the darts land inside the bulls-eye. Find σ (measured in inches).

(ii) Find the expected distance of the dart from the center of the dartboard.

(Hint: Use polar coordinates.)

Graduate problems: Chapter 6, Theoretical exercises: #11, 18.