Spring, 2014 — T. Shifrin

MATH 4250/6250
PROBLEM SET #4

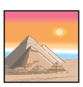
DUE February 19, 2015.

Instructions: Turn in (at least) five problems of your choice, but **including any underlined problem(s)**. Graduate students should include at least one "pyramid" problem.

Problems to work but not hand in:
§2.1: #1, 2, 3a,d, 4a, 5, 7.


§2.1: #3b **and** c, 4b **and** c, 6, <u>**10**</u>.


§2.1: #11, 12[1], 13, 16[2].


§2.1: #14, 17.

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[1]If you have the published version from Baxter St. Books, this problem has been updated as follows: Given a ruled surface M parametrized by $\mathbf{x}(u,v) = \boldsymbol{\alpha}(u) + v\boldsymbol{\beta}(u)$ with $\boldsymbol{\alpha}' \neq 0$ and $\|\boldsymbol{\beta}\| = 1$.

(a) Check that we may assume that $\boldsymbol{\alpha}'(u)\cdot\boldsymbol{\beta}(u) = 0$ for all u. (Hint: Replace $\boldsymbol{\alpha}(u)$ with $\boldsymbol{\alpha}(u)+t(u)\boldsymbol{\beta}(u)$ for a suitable function t.)

(b) Suppose, moreover, that $\boldsymbol{\alpha}'(u)$, $\boldsymbol{\beta}(u)$, and $\boldsymbol{\beta}'(u)$ are linearly dependent for every u. Conclude that $\boldsymbol{\beta}'(u) = \lambda(u)\boldsymbol{\alpha}'(u)$ for some function λ. Prove that:
  (i) If $\lambda(u) = 0$ for all u, then M is a cylinder.
  (ii) If λ is a nonzero constant, then M is a cone.
  (iii) If λ and λ' are both nowhere zero, then M is a tangent developable. (Hint: Find the directrix.)

[2]You will need to use `Mathematica` for part c of this problem. It would be particularly enlightening to have `Mathematica` plot on a single set of axes the area of two disks and the area of the catenoid(s) as functions of R. To have `Mathematica` find the root of an equation, you use the `FindRoot` command. Moreover, if you want to solve $f(x) = R$ (starting Newton's method off at a value $x = x_0$) and name the solution $x(R)$ as a function of R, you can write `x[R_]:=x/.FindRoot[f[x]==R,{x,x_0}]`. Note that if there's more than one solution, changing the initial value x_0 may lead you to different solutions. There's more information on this and a pertinent example in the primer.