Spring, 2015 T. Shifrin

MATH 3510(H)
PROBLEM SET #13

DUE Friday, April 24, 2015.

§9.3: #2, 4, 11a,d, 12a,d, 13c, 15.

§9.4: #6, 7, 8, 9.

Problems to turn in:

WeBWork Homeworks 13 and 14

§9.2: #15 (2), 17 (2), 19 (4), 22 (3), 24* (4).

> **A.** (2) Use #22 to prove that a non-diagonalizable $n \times n$ matrix with real eigenvalues is arbitrarily close to a diagonalizable matrix.

§9.3: #17a,b† (3), 18 (3), 21 (2).

§9.4: #7 (2), 8 (2), 9 (2), 15 (3).

Challenge problems (Turn in separately):

§9.2: #20 (4), 21 (3).

> **B.** (3) Suppose A is a 2×2 matrix. Suppose λ is its only eigenvalue, having algebraic multiplicity 2 but geometric multiplicity 1. Prove that there is a basis for $\mathbb{R}^2$ with respect to which the matrix of A is $\begin{bmatrix} \lambda & 1 \\ & \lambda \end{bmatrix}$. (This is a special case of the Jordan canonical form.) Hint: Show that $\mathbf{C}(A-\lambda I) \subset \mathbf{N}(A-\lambda I)$ and therefore $\mathbf{C}(A-\lambda I) = \mathbf{N}(A-\lambda I)$.

§9.4: #11 (3), 14 (3), 21 (4), 22 (4).

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*Don't forget the hint for part (c) at the top of p. 436.

†As the hint suggests, consider $\mathbf{f}(t) = e^{tA}e^{-tA}$ and compute $\dot{\mathbf{f}}(t)$.