Spring, 2015 **MATH 3510(H)** T. Shifrin

PROBLEM SET #11

DUE Wednesday, April 8, 2015.

Problems to work but not hand in:

§8.6: #1, 7.

Problems to turn in:

WeBWork Homework 11

§8.6: #2 (3), 8 (3), 9 (3), 10 (3).

§8.7: #5 (3), 6* (3), 7 (3).

A. (3) Suppose $\mathbf{f}\colon S^n \to S^n$ has no fixed point. Show that $\mathbf{f}$ is homotopic to the antipodal map $\mathbf{g}(\mathbf{x}) = -\mathbf{x}$.† Conclude that if $\omega \in \mathcal{A}^n(S^n)$ is the usual volume form, then $\displaystyle\int_{S^n} \mathbf{f}^*\omega = (-1)^{n+1}\int_{S^n}\omega$.

B. (3) Suppose $\mathbf{f}\colon S^n \to S^n$ is homotopic to a constant map. Prove that there are $\mathbf{x}, \mathbf{y} \in S^n$ with $\mathbf{f}(\mathbf{x}) = \mathbf{x}$ and $\mathbf{f}(\mathbf{y}) = -\mathbf{y}$. (Hint: Proceed by contrapositive, being careful with your negation of the conclusion; apply **A** and an obvious cousin.)

Challenge problems (Turn in separately):

§8.6: #6‡ (4), 11 (3), 13 (4), 14 (3).

§8.7: #8 (3), 9 (4), 12 (4), 14 (2), 15 (4), 16 (6).

*Hint: Create a $(k-1)$-form $\omega \in \mathcal{A}^{k-1}(\partial M)$ by using the proof of Theorem 5.1 to find a *single* function ρ on a *single* coordinate chart on ∂M with nonzero integral.

†Hint: What do you know about the line segment joining $\mathbf{f}(\mathbf{x})$ and $-\mathbf{x}$?

‡The formula (†) appears on p. 396.