Fall, 2014

MATH 3500(H)
PROBLEM SET #7

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DUE Wednesday, October 8, 2014.

Problems to work but not hand in:

§3.3: #1, 2, 5, 16.

§3.4: #1a, 2b, 3, 4.

Problems to turn in:

WeBWork Homework 7

§3.2: #16* (3), 18 (3).

A. If the partial derivatives of f are bounded on $U \subset \mathbb{R}^2$, prove that f is continuous.

§3.3: #11† (3), 15 (3).

B. (3) Suppose $f\colon \mathbb{R}^2 \to \mathbb{R}$ is differentiable and we define $F\begin{pmatrix} u \\ v \end{pmatrix} = f\begin{pmatrix} e^u \cos v \\ e^u \sin v \end{pmatrix}$. Use the chain rule carefully to compute $\left(\dfrac{\partial F}{\partial u}\right)^2 + \left(\dfrac{\partial F}{\partial v}\right)^2$ in terms of $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$, evaluated at the appropriate spot.

§3.4: #5‡ (3), 6 (2).

C. (3) Suppose $f\colon \mathbb{R}^2 \to \mathbb{R}$ is differentiable and $y^2\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} = 0$ everywhere. Find the level curves of f. (Hint: From knowing the direction of the gradient, you should figure out the slope of the level curve through $\begin{bmatrix} x \\ y \end{bmatrix}$ and solve the differential equation $dy/dx = \dots$.) Deduce that if we know f on the x-axis, then we know f everywhere in the plane: In particular, if $f\begin{pmatrix} x \\ 0 \end{pmatrix} = F(x)$, give a formula for f.

Challenge problems (Turn in separately):

§3.2: #19 (5).

D. (4) Give a criterion (in terms of α, β, γ, and δ) for the function in §2.3, #17 to be differentiable at $\mathbf{0}$.

E. Let $U \subset \mathbb{R}^2$ be a neighborhood of $\mathbf{0}$ and $f\colon U \to \mathbb{R}$ is continuous. Suppose both partials $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}$ exist at $\mathbf{0}$ and *only* the former is continuous. Prove that f is differentiable at $\mathbf{0}$.

§3.4: #7 (3), 8 (3), 9 (3), 13 (4), 15 (4).

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*Hint: Copy the (beginning of the) framework of the proof of Prop. 2.4.

†Please add the hypothesis that f is differentiable. Also, contemplate to which WeBWork problem this result might be relevant.

‡Set this up explicitly as a chain rule problem, and apply the results of Example 3.