Fall, 2014 | **MATH 3500(H)** | T. Shifrin

PROBLEM SET #6

DUE Wednesday, October 1, 2014.

Problems to work but not hand in:

§3.1: #1a,c, 2a,b, 3a, 11.

§3.2: #1a, 2a, 3a,b,d, 4.

Problems to turn in:

WeBWork Homework 6

§2.3: #7 (3), 11 (2), 15 (3).

§3.1: #4 (1), 5 (2), 8* (3), 13b (2).

§3.2: #5 (2), 9 (3), 17 (3).

A. (3) Show that for any $\mathbf{a} \neq \mathbf{0}$ we have

$$\lim_{\mathbf{h}\to\mathbf{0}} \frac{\|\mathbf{a}+\mathbf{h}\| - \left(\|\mathbf{a}\| + \dfrac{\mathbf{a}\cdot\mathbf{h}}{\|\mathbf{a}\|}\right)}{\|\mathbf{h}\|} = 0\,.$$

(Hint: Remember that $x - y = \dfrac{x^2-y^2}{x+y}$. This is the same "conjugate trick" you used numerous times in single-variable calculus.) If $f(\mathbf{x}) = \|\mathbf{x}\|$ and $\mathbf{a} \neq \mathbf{0}$, why have you shown that f differentiable at $\mathbf{a}$? What is $Df(\mathbf{a})$?

Challenge problems (Turn in separately):

§2.3: #16 (5), 17 (5).

B. (4) Prove or give a counterexample: If $f\colon \mathbb{R}^2 \to \mathbb{R}$ is continuous along every (continuous) path $\mathbf{g}$ through the origin, then f is continuous at the origin.

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*Don't forget the single-variable chain rule!