Fall, 2014 — T. Shifrin

MATH 3500(H)
PROBLEM SET #3

DUE Wednesday, September 10, 2014.

Problems to work but not hand in:

§1.5: #1, 2, 5a, 6a, 7a, 8, 12, 15.

§2.1: #1b,e.

Problems to turn in:

WeBWork Homework 3

§1.4: #34 (4), 36 (4).

A. (4) The goal of this problem is to prove without explicit matrix entries that the rotation matrix $A = A_\theta$ satisfies $A^{-1} = A^{\mathsf{T}}$.

(i) Suppose B is an $m \times n$ matrix and $B\mathbf{x} \cdot \mathbf{y} = 0$ for all $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{y} \in \mathbb{R}^m$. Prove that $B = \mathsf{O}$.

(ii) Deduce that if C and D are $m \times n$ matrices and $C\mathbf{x} \cdot \mathbf{y} = D\mathbf{x} \cdot \mathbf{y}$ for all $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{y} \in \mathbb{R}^m$, then $C = D$.

(iii) Let $A = A_\theta$ be the usual rotation matrix. Using the geometric interpretation of dot product (and not the actual matrix entries), show that $A\mathbf{x} \cdot \mathbf{y} = \mathbf{x} \cdot A^{-1}\mathbf{y}$ for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$. Use part (ii) to deduce that $A^{-1} = A^{\mathsf{T}}$.

§1.5: #10 (3), 13(3), 17* (5).

§2.1: #2a† (2), 3 (3), 11 (3).

Challenge problems (Turn in separately):

§1.5: #3 (3), 18 (3), 19 (4), 20 (2).

---

*Further hint for (a): Since we're treating $\mathbf{u}$ as the tail of our vectors, don't forget to be consistent. Comment for (b): Given the set-up for (a), it'll be easier to show that t is the ratio of the signed area of $\triangle\mathbf{uvx}$ to the signed area of $\triangle\mathbf{uvw}$. Hint for (d): Apply the results of (a) and (b) to $\mathbf{x} = \mathbf{0}$.

†Note that in **(a)** we parametrize the circle *except* for the point $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$.