Fall, 2014

MATH 3500(H)
PROBLEM SET #11

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DUE Wednesday, November 5, 2014.

Problems to work but not hand in:

§4.5: #3, 6, 11.

§5.1: #1.

Problems to turn in:

WeBWork Homework 11

§4.4: #16 (4).

> **A.** (3) Let A be an $m \times n$ matrix with $\dim \mathbf{N}(A) = k$. Suppose $W \subset \mathbb{R}^m$ is an ℓ-dimensional subspace. You can easily check (but I won't ask you to) that $V = \{\mathbf{x} \in \mathbb{R}^n : A\mathbf{x} \in W\}$ is subspace.*
>
> (i) Show that if $W \subset \mathbf{C}(A)$, then $\dim V = k + \ell$. (Hint: Give a basis for V.)
>
> (ii) Conclude, more generally, that $k \leq \dim V \leq k + \ell$. (Hint: Consider $W \cap \mathbf{C}(A)$.)

§4.5: #2a,b,e (3), 4 (3), 5 (2), 9 (2).

§5.1: #2† (3), 8 (3).

Challenge problems (Turn in separately):

§4.4: #18 (4).

§5.1: #10 (5), 12 (3), 13 (4), 14 (2).

*If $A = [T]$, then $V = T^{-1}(W)$.

†Hint: See Exercise 2.3.2. Since there are two ways X can fail to be compact, you will need to give a function f for each of those cases.