Math 3100H – Learning objectives to meet for Exam #1

The exam will cover §§1.1–1.5 of the course notes, with an emphasis on §§1.1–1.4. §1.6 is not examinable.

What to be able to state

Basic definitions

You should be able to give complete and precise definitions of each of the following items:

• sequence, terms of a sequence, graph of a sequence
• increasing, decreasing, monotone, and the “strictly” and “eventually” variants
• bounded above, bounded below, upper bound, lower bound, bounded
• subsequence
• \{a_n\} converges to \(L\), where \(L\) is a real number
• \{a_n\} diverges
• geometric sequence
• continuous function
• \(\lim_{x \to \infty} f(x) = L\), where \(L\) is a real number
• \(\lim_{x \to \infty} f(x) = \infty\)
• \(\lim_{n \to \infty} a_n = \infty\)

Big theorems

Give full statements of each of the following results, making sure to indicate all necessary hypotheses. For results proved in class, describe the components and main ideas of the proof.

• Principal of mathematical induction, complete mathematical induction
• A convergent sequence has a **unique** limit
• Convergent sequences are bounded
• Subsequences of convergent sequences converge to the original limit
• Behavior of geometric sequences (Proposition 1.4.15)
• If \{a_n\} is bounded above by \(U\) and \(a_n \to L\), then \(L \leq U\)
• (Bounded) \cdot (going to 0) goes to 0
• Sum rule for limits, product rule for limits, quotient rule for limits
• Continuous functions “commute” with limits (Proposition 1.5.10)
• If \(\lim_{x \to \infty} f(x)\) exists or diverges to \(\infty\), then \(\lim_{n \to \infty} f(n)\) exhibits the same behavior
• L’Hôpital’s rule
What to expect on the exam

You can expect five to six questions on the exam, including

- one problem testing mathematical induction
- one problem testing your ability to compute a limit directly from the $\epsilon$-$N$ definition

The rest of the exam is designed to test your comfort level working with the basic definitions. I am not interested in having you regurgitate proofs of results from the notes; I want to know if you have internalized the ideas enough to solve similar problems.