PROF. CLARK'S MATH 3200 FALL 2009 MIDTERM 1

Part I: Do all of the following problems.

I.1) a) Show that the following are logically equivalent:
(i) $A \implies (B \vee C)$;
(ii) $(A \wedge \sim B) \implies C$.
b) Show that for all $x \in \mathbb{Z}$, $x(x^2+1)$ is even.

I.2) Negate the following statements:
a) x is odd and if y is even, then z is odd.
b) For any line ℓ in the plane and any point P not lying on ℓ, there exists exactly one line ℓ' passing through P and parallel to ℓ.
c) I'm not going to lower my voice, and I'm staying right where I am.
d) All's well that ends well.

I.3) Consider an implication of the form "$\forall x \in S, P(x) \implies Q(x)$."
a) What does it mean for the implication to hold trivially?
b) What does it mean for the implication to hold vacuously?
c) Let $S = \mathbb{Z}$. Let $P(x)$ be "$1298548x + 1509850980$ is odd", and let $Q(x)$ be "$191 \mid x^4 + 53x^3 + 17x^2 - 14$." Show that for all $x \in S$, $P(x) \implies Q(x)$.
d) Let S be the set of rational numbers. Let $P(x)$ be "e^x is an irrational number." Let $Q(x)$ be "$x^4 + 1 \geq 2x^2$." Show that for all $x \in S$, $P(x) \implies Q(x)$.

I.4) Consider an implication of the form "$\forall x \in S, P(x) \implies Q(x)$."
a) What does it mean for the implication to hold trivially?
b) What does it mean for the implication to hold vacuously?
c) Let $S = \mathbb{Z}$. Let $P(x)$ be "$1298548x + 1509850980$ is odd", and let $Q(x)$ be "$191 \mid x^4 + 53x^3 + 17x^2 - 14$". Show that for all $x \in S$, $P(x) \implies Q(x)$.
d) Let S be the set of rational numbers. Let $P(x)$ be "e^x is an irrational number". Let $Q(x)$ be "$x^4 + 1 \geq 2x^2$". Show that for all $x \in S$, $P(x) \implies Q(x)$.

Part II: Do **any two of the following three** problems.[1]

II.1) Prove or disprove:
a) If x, y, z are objects such that $x \in y$ and $y \in z$, then $x \in z$.
b) If X, Y, Z are sets such that $X \subseteq Y$ and $Y \subseteq Z$, then $X \subseteq Z$.
c) If X, Y, Z are sets such that $X \subsetneq Y$ and $Y \subsetneq Z$, then $X \subsetneq Z$.

II.2) Let A and B be sets. Show that $A \cap B = A \cup B$ if and only if $A = B$.

II.3) Show that for all $x \in \mathbb{Z}$, $8 \mid (x-1)(x-2)(x-3)(x-4)$.

[1] If it is not clearly indicated which two I am meant to grade, I will simply grade the first two problems that have nonempty answers.