<u>Section 2.2</u>: Circles, Intercepts and Symmetry

From last class, we saw the circle with center (h, k) and radius r is the collection of points (x, y) at distance r from (h, k). The Distance Formula produces: $\sqrt{(x-h)^2 + (y-k)^2} = r$. By squaring the Distance Formula, you obtain:

Formula (Standard Form of Circles)**.**

$$(x-h)^2 + (y-k)^2 = r^2$$

With circles, y is NOT a function of x, which makes circles harder to graph on a calculator. That's why we can also break up circles into two **semicircles** (the top and bottom halves). To get these, solve for y in terms of x. Here's one special case:

Formula (Semicircles centered at origin)**.** *By solving $x^2 + y^2 = r^2$ for y, you get*

$$y = \sqrt{r^2 - x^2} \text{ (top half) or } y = -\sqrt{r^2 - x^2} \text{ (bottom half)}$$

EX 1: Find the centers and radii of the following (semi)circles:
(a) $(x-2)^2 + (y+1)^2 = 5$ (b) $y = -\sqrt{16 - x^2}$

EX 2: Find equations for the circles with these properties:

1. The center is $(0, 1)$, and $(1, 2)$ is on the circle.
2. A diameter has endpoints $(1, 1)$ and $(-2, -3)$.
3. The circle is tangent to both axes, has radius 4, and has its center in Quadrant III.

Intercepts

The x**-intercepts** of a graph (also called its *zeroes* or *roots*) are the values where the graph touches the x-axis. To find them, set $y = 0$ and solve for x. Similarly, the *y-intercepts* are where the graph touches the x-axis, so you set $x = 0$ to find them.

For instance, to get the x-intercept of the line $y = 2x + 1$, set $y = 0$ and get $0 = 2x + 1$. Solving for x gives $2x = -1$, so $x = -1/2$.

EX 3: Find the x-intercepts of the circle with center $(-3, 4)$ and radius 5. (In other words, you're finding all values x such that $(x, 0)$ is on the circle.) Also find the y-intercepts!

Completing the Square

You may occasionally see a circle presented in the *general form*

$$x^2 + ax + y^2 + by + c = 0$$

where a, b, c are constants. To get this in standard form, we use *completing the square.* First, I move the c over by subtracting it to both sides. Next, when you see $x^2 + ax$, you add $(a/2)^2$ to both sides of the equation. When you do this, you get

$$x^2 + ax + (a/2)^2 \text{ which becomes } (x + a/2)^2$$

After doing this for x and y, move the constants to the other side.

EX 4: Find the centers and radii of these circles by completing the square:
(a) $x^2 - 2x + y^2 - 3 = 0$ (b) $x^2 + 4x + y^2 + 6y = 12$

Symmetry

Some kinds of symmetry in a graph are easy to check from its equation. To see if a graph has symmetry, you perform a change to its equation and see if the (simplified) equation stays the same!

Symmetry Type	How to test for it	A couple examples	
With respect to (w.r.t.) y-axis	Replace x by $-x$	$y = x^2$	$y = \|x\|$
W.r.t. x-axis	Replace y by $-y$	$x = y^2$	$y = 0$
Rotation w.r.t. origin	Replace x by $-x$ AND y by $-y$	$y = x^3$	$x^2 + y^2 = 1$

EX 5: Find out what kinds of symmetry these graphs have.
(a) $y = x^4 + x^2$ (b) $y = 1/x^3$ (c) $y = \sqrt{x}$ $y = \frac{1}{x-1}$

Section 2.3: Lines

Lines are curves that "never change direction". We make this more formal with the concept of **slope**. If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points, the slope from A to B is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ a.k.a. } \frac{\Delta y}{\Delta x} = \frac{\text{"rise"}}{\text{"run"}}$$

m measures how steep the line through A and B is. When $m > 0$, the line rises. When $m < 0$, the line falls.

Two special cases are very important.

- A **horizontal line** has zero slope, $m = 0$. (We also call a horizontal line a *constant function*, since y never changes its value.) Its equation has the form $y = b$.
- A **vertical line** has undefined slope, so m DNE. Its equation has the form $x = a$.

A line which is not horizontal or vertical is called **oblique**.

EX 6: Find equations for:

(a) The vertical line through $(2, 3)$ (b) The horizontal line through $(0, 5)$

Line Formulas

There are two very popular ways of writing lines, each with their own advantages. The main ingredient in both of these formulas is the slope m, so usually you compute that first!

Formula (Point-Slope Form)**.** *The line with slope m through $P(x_1, y_1)$ has equation*

$$y - y_1 = m(x - x_1)$$

This comes from writing the slope equation: $(y - y_1)/(x - x_1) = m$, and then multiplying $x - x_1$ to the other side.

Formula (Slope-Intercept Form)**.** *The line with slope m and y-intercept b has equation*

$$y = mx + b$$

Usually, point-slope form is easier to make from the information you're given. You can always convert it to slope-intercept form by solving for y. Slope-intercept form, however, can be easier to graph and use when solving equations.

EX 7: Find equations for the following lines.

1. The line through $A(7, 0)$ and $B(-9, 2)$
2. The line through $(3, 2)$ with x-intercept 7
3. The line through the centers of the circles $x^2 + (y - 1)^2 = 4$ and $(x - 3)^2 + (y + 1)^2 = 1$

Another kind of equation you may see is the **standard form of a line**

$$ax + by = c$$

where a and b are not both 0. If b is not 0, you can solve for y and get an equation in slope-intercept form. If b is 0, solve for x instead (vertical line).

EX 8: Find the slopes and y-intercepts of the following lines.

(a) $2x + 3y = 4$ (b) $x - 5y = 1$

Parallel and Perpendicular

Two **parallel** ($\|$) lines have the same slope. If two different lines are parallel, their graphs never intersect. (Note: a line IS considered parallel to itself!)

Two **perpendicular** ($\perp$) lines have slopes which are negative reciprocals. Thus, if m_1 and m_2 are the two slopes, then $m_2 = -1/(m_1)$. (The one exception is that horizontal and vertical lines are perpendicular, clearly.)

EX 9: Find an equation for the line through $(1, 3)$ perpendicular to $2x + y = 5$.