Math 3100 Assignment 6
More Infinite Series

Due at 12:00 pm on Friday the 2nd of March 2018

1. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of non-negative terms. Use the Monotone Convergence Theorem to prove that $\sum_{n=1}^{\infty} a_n$ converges if and only if the sequence of partial sums is bounded.

   Be sure to prove both implications

2. Let $a_n \geq 0$ for all $n \in \mathbb{N}$.

   (a) Show that if $\lim_{n\to\infty} na_n$ exists and is not equal to 0, then $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.

   (b) Show that if $\lim_{n\to\infty} n^2 a_n$ exists, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

3. Determine which of the following series converge, and which diverge. Give reasons for your answer.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n - 1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\log n}{n^2}$ (c) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{\sqrt{n \log n}}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(1+n^2)^{1/3}}{n}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(1+n^2)^{1/3}}{n^2}$

4. Determine which of the following series are absolutely convergent, which are conditionally convergent, and which diverge. Give reasons for your answer.

   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n\sqrt{n}}$ (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n n}{(n+1)^5}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n+1}}{n(-3)^n}$ (e) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n!}{(2n)!}$