Math 3100 Assignment 5
Infinite Series

Due at 12:00 pm on Monday the 26th of February 2018

1. Suppose that $\sum_{k=1}^{\infty} a_k$ converges to A and $\sum_{k=1}^{\infty} b_k$ converges to B.

   (a) Prove that $\sum_{k=1}^{\infty} (a_k + b_k)$ converges to $A + B$.

   (b) Must $\sum_{k=1}^{\infty} (a_k b_k)$ converge to AB? Give either a proof or counterexample.

2. Evaluate the following series

$$\text{(a)} \quad \sum_{n=1}^{\infty} \frac{1}{2^n} \qquad\qquad \text{(b)} \quad \sum_{n=2}^{\infty} \frac{3}{4^n} \qquad\qquad \text{(c)} \quad \sum_{n=3}^{\infty} \frac{7^{n-1}}{2^{n+1}}$$

3. Prove that omitting or changing a finite number of terms of a series does not affect its convergence.
   Hint: Try using the Cauchy Criterion

4. Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be two sequences of positive real numbers. Prove the following:

   (i) If $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$, then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or both diverge.

   (ii) If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ also converges.

   (iii) If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ also diverges.

5. Test the series for convergence or divergence.

$$\text{(a)} \ \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+3} \quad \text{(b)} \ \sum_{n=0}^{\infty} \cos(n) \quad \text{(c)} \ \sum_{n=1}^{\infty} \frac{2^n}{n3^{n+1}} \quad \text{(d)} \ \sum_{n=1}^{\infty} \frac{n2^n}{3^{n+1}} \quad \text{(e)} \ \sum_{n=3}^{\infty} \frac{(-1)^n}{(\log n)^2}$$

$$\text{(f)} \ \sum_{n=1}^{\infty} \frac{2n}{8n-5} \quad \text{(g)} \ \sum_{n=3}^{\infty} \frac{2}{n(\log n)^3} \quad \text{(h)} \ \sum_{n=1}^{\infty} \frac{3^n n^2}{n!} \quad \text{(i)} \ \sum_{n=1}^{\infty} \frac{3^n}{5^n+n} \quad \text{(j)} \ \sum_{n=1}^{\infty} \frac{n+5}{5^n}$$

6. Investigate the behavior (convergence or divergence) of $\sum_{n=1}^{\infty} a_n$ if

$$\text{(a)} \ a_n = \sqrt{n+1} - \sqrt{n} \qquad\qquad \text{(b)} \ a_n = \frac{\sqrt{n+1} - \sqrt{n}}{n}$$