Math 2270 **Spring 2019**

Old Exam 3

No calculators are allowed

1. (30 points) Let C denote the portion of a spiral that is given by the vector equation
$$\mathbf{r}(t) = 2t\,\mathbf{i} + 3t^2\,\mathbf{j} + 3t^3\,\mathbf{k}, \quad 0 \le t \le 2.$$
   (a) Evaluate the line integral
$$\int_C x\,ds.$$
   Hint: $(2+9t^2)^2 = 4 + 36t^2 + 81t^4$

   (b) Calculate the work done by the force field
$$\mathbf{F}(x,y,z) = z\,\mathbf{i} + y\,\mathbf{j} + x\,\mathbf{k}$$
   in moving a particle from the point $(0,0,0)$ to the point $(4,12,24)$ along the curve C in two ways:

   i. Using the given parameterization to evaluate the work integral

   ii. By evaluating the potential function for $\mathbf{F}$

2. (15 points) Use **Green's Theorem** to evaluate
$$\int_C xy\,dx + x^2\,dy$$
where C is the triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$.

3. (10 points) Set up, **but do not evaluate**, a double integral in polar coordinates that is equal to the surface integral
$$\iint_S xz\,d\sigma$$
where S is the portion of the paraboloid $z = x^2 + y^2$ that lies between the planes $z = 1$ and $z = 4$.

4. (30 points) Let $\mathbf{F} = \langle 2y, z, 5x \rangle$ and C denote the intersection of the plane $y + z = 3$ and the cylinder $x^2 + y^2 = 5$.

   (a) Find the flux
$$\iint_S \mathbf{F} \cdot \mathbf{n}\,d\sigma$$
   of the vector field $\mathbf{F}$ upwards across S, where S is the portion of the plane which lies inside C.

   (b) Using **Stokes' theorem** to evaluate the circulation
$$\int_C \mathbf{F} \cdot d\mathbf{r}$$
   of the vector field $\mathbf{F}$ counterclockwise (as viewed from above) around C.

5. (15 points) Use the **Divergence Theorem** to evaluate
$$\iint_S \mathbf{F} \cdot \mathbf{n}\,d\sigma$$
where $\mathbf{F} = \langle y, x, z \rangle$ and S is the boundary of the solid region D enclosed by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$.