

Exam 1

1. (8 points) Solve the following two initial value problems

(a) $\frac{dy}{dx} = \frac{\ln(2x+1)}{2x+1}$, $y(0) = 1$

(b) $\frac{dy}{dx} + 2xy = 0$, $y(0) = 2$

2. (8 points) Let \mathcal{C} denote the portion of the curve given by

$$y = x^2 - \frac{1}{8} \ln x, \quad 1 \leq x \leq e$$

- (a) Find the length of the curve \mathcal{C} .
- (b) Find the area of the surface generated by rotating the curve \mathcal{C} about the line $x = -2$.
3. (12 points) Let \mathcal{R} denote the region in the plane bounded between the curves $y = 2 - x$ and $x = y^2$. Set up, **but do not evaluate**, a definite integral equal to the volume of
- (a) the solid region obtained by revolving \mathcal{R} about the line $y = 2$
- (b) the solid region obtained by revolving \mathcal{R} about the line $x = 4$
- (c) the solid region with base \mathcal{R} whose cross-sections perpendicular to the y -axis are isosceles right triangles with hypotenuse in the base.
4. (6 points) An cylindrical aquarium of radius 1 m and height 4 m is full of water. Find the work required to pump half of the water out of the aquarium.
(You may assume that 1 cubic meter of the water weighs 10,000 N)
5. (8 points) A uniform cable hanging over the edge of tall building is 40 ft long and weighs 80 lb. Find the work required to pull the 10 ft of the cable closest to top of the building to the top?
(You may assume that the building is taller than 40 ft)
6. (8 points) If 4 J of work are needed to stretch a spring from 3 m to 4 m and another 7 J are needed to stretch it from 4 m to 5 m, what is the natural length of the spring?