Math 2260 **Spring 2017**

Practice Exam 3

1. (10 points) Evaluate the following limits, be sure to state any special limits or theorems that you use.

$$\text{(a)} \lim_{n\to\infty} \frac{\sin n}{\sqrt{n}} \qquad \text{(b)} \lim_{n\to\infty} \sqrt{\frac{n^2+\ln n}{4n^2+1}}$$

2. (10 points) Determine which of the following series converge or diverge. Justify your answers.

$$\text{(a)} \sum_{n=0}^{\infty} \frac{n+1}{2^n} \qquad \text{(b)} \sum_{n=0}^{\infty} (-1)^n \frac{n^3}{2n^3+1}$$

3. (15 points)

   (a) Find the *radius of convergence* and *interval of convergence* of the power series

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$$

   (b) Find the sum of the series above for those values of x that lie strictly inside the series interval of convergence.

4. (10 points)

   (a) Write down the Maclaurin series for $f(x) = \tan^{-1} x$.

   (b) Evaluate

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3^n(2n+1)}$$

5. (25 points)

   (a) Write down the Maclaurin series for $f(x) = \ln(1+x)$ and use this to evaluate

$$\lim_{x\to 0} \frac{\ln(1+x)}{x}$$

   (b) Use your answer to Part (a) to evaluate

$$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$$

   (c) Use your answer to Part (a) to determine if the following series are convergent or divergent.

$$\text{(i)} \sum_{n=1}^{\infty} \ln(1+n^{-3}) \qquad \text{(ii)} \sum_{n=1}^{\infty} n^2 \ln(1+n^{-3})$$

6. (10 points) Find $f^{(8)}(0)$ when $f(x) = (1+3x^2)^{-1}$ without differentiating.

7. (10 points) For what values of x does $1-x^2/2$ approximate $\cos x$ to within 0.1?

8. (10 points) Find a polynomial which approximates e^x to within 0.01 for all $|x| \le 1/2$.