

MATH 4780/6780: MATHEMATICAL BIOLOGY

Online Assignment 4

The assignment is due **Tuesday 4/14 by 9pm**. Please e-mail your solution to ocaner@uga.edu with Subject line “online assignment”.

1. Describe the long term behavior of the SIS model. In other words
 - (a) Derive a single variable ODE using conservation.
 - (b) Find all fixed points.
 - (c) Draw the phase line.
 - (d) Describe the long term behavior with a sentence.
2. Analyze the SIR model using XPP.

$$\begin{aligned}S' &= -\beta IS \\I' &= I(\beta S - \gamma)\end{aligned}$$

Assume that S , I and R represent the ratio of the susceptible, infected and removed individuals in the entire population. In other words, $S(t) + I(t) + R(t) = 1, \quad \forall t$. Assume that initially only one percent of the population is infected, that is, $S(0) = 0.99, I(0) = 0.01, R(0) = 0$. Let the infection rate $\beta = 1$ and recovery rate $\gamma = 0.1$.

- (a) Using this information, write an XPP ODE file.
- (b) Using XPP, draw the nullclines, the scaled direction field, and the solution. Include a screenshot.
- (c) Approximately what percent of the population is infected when the infection is at its peak?
- (d) Answer the same question in (c) when you change the infection rate from $\beta = 1$ to $\beta = 3$. And also for $\beta = 0.2$.
- (e) Answer the same question in (c) when you change the recovery rate from $\gamma = 0.1$ to $\gamma = 0.3$ (while keeping the infection rate $\beta = 1$). And also for $\gamma = 0.02, \gamma = 0.5$ and $\gamma = 1$.