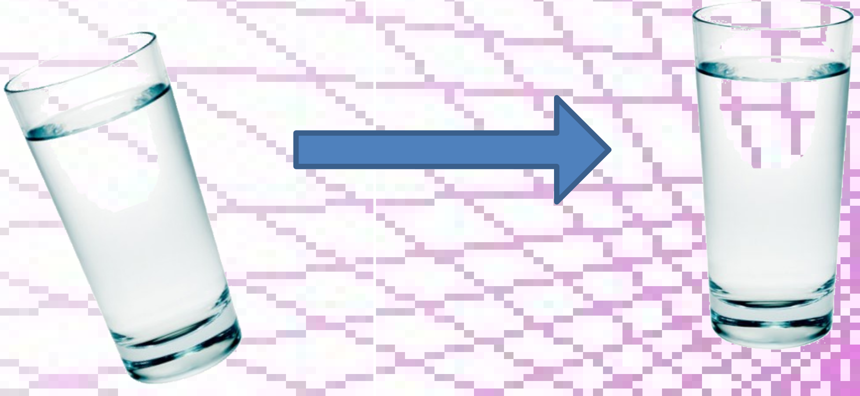
Tipping Points in an Open System

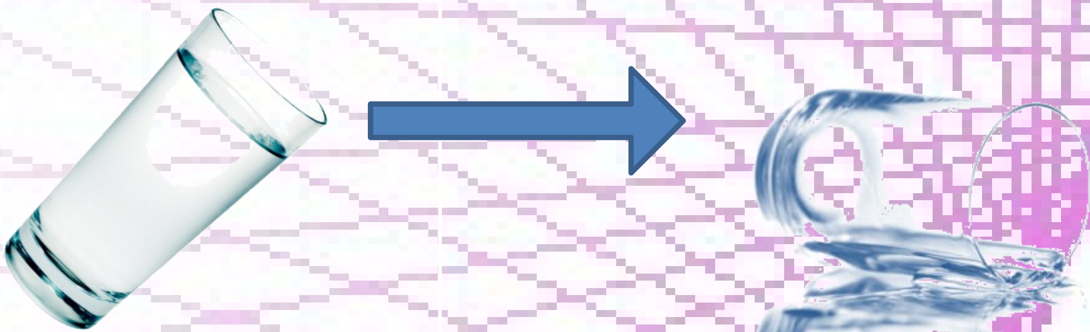
Tipping Points

- A tipping point is a state where small perturbations can cause the graph to go to another stable state.
- A simple example of a tipping point is that of a glass of water. If a glass tilted as below, it will right itself, i.e. go back to the original stable state.



Tipping Points

- However, if the glass is tipped too far (passed the tipping point), it will fall over.
- In the case of climatology, if a climate (overall weather) is “pushed” far enough from its original stable state that it passes the tipping point, it could go to a different steady state, and be possibly irreversible

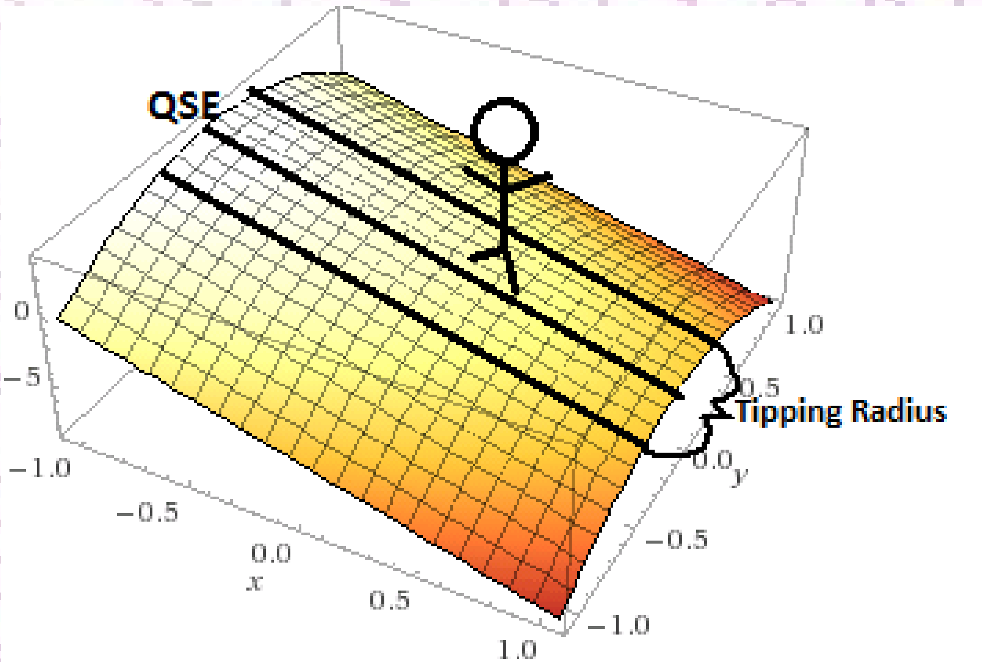


Terms

- Quasi-static Equilibrium (QSE)
  - The steady state of a system.
- Tipping radius
  - The radius around a system inside which it will move along the QSE. Outside of which, it will tip to a different path

Explanation of Terms

- Imagine walking up a mountain. As long as you stay near the path, you won't fall off. However, if you stray too far, you will.
- In this example:
  - Path: QSE
  - "Near": Tipping radius



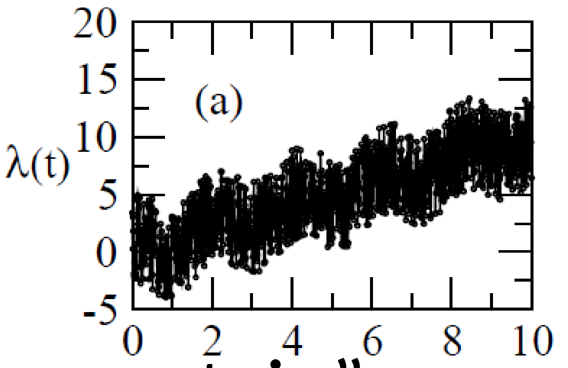
Open Systems

- An open system is where the model can be influenced by outside forces.
- Tipping Points can only occur in open systems, because if left alone, a model will continue in its initial steady state

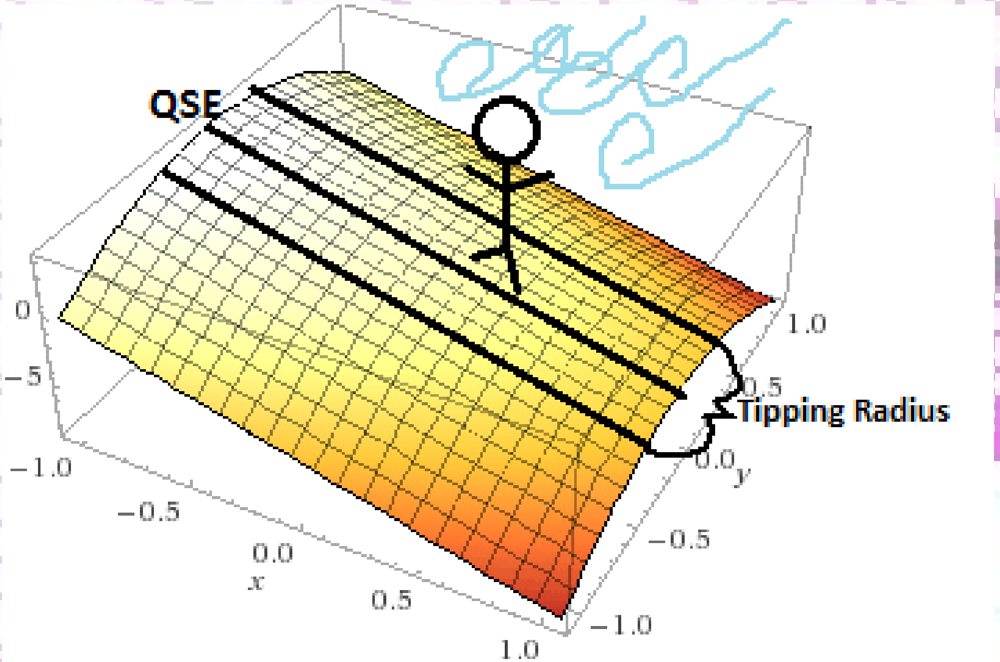
Types of Tipping Points

- "B-tipping": Where the system tips due to a bifurcation
- "N-tipping": Where the system tips due to noise
- "R-tipping": Where the system tips simply due to its rate of change

“N-tipping”



- Referring back to the “Climbing a mountain” illustration, if you were to try and climb the mountain with unsteady wind, then you may experience N-tipping, where a gust might push you out of the tipping radius



"B-tipping"

- Using the same illustration, if the path you are using to climb the mountain splits, then you are at B-tipping, where a bifurcation tips you away from where you were going

“R-tipping”

- R-tipping occurs when the QSE is traveling at such a rate that the graph cannot keep up with it, and tips to another path.
- In the illustration, based on the initial condition, the graph should follow f(t), but f(t) moved too quickly, so it follows g(t)

A Tipping Point in the Climate

- One tipping point currently hypothesized to trigger in the next 50 years is forest dieback in the Amazon and the Boreal. Dieback is where many trees in a forest die due to increased heat and drought (among other factors). Currently, this dieback is under control and not affecting the overall climate, but if temperature continues to rise, dieback will continue and CO_2 levels would rise as the trees would be unable to convert it to O_2, which would cause the temperature to rise and dieback to continue.

Open System in Forest Dieback

- In the example of forest dieback, deforestation by humans and the release of CO_2 and other greenhouse gasses by humans are external factors that influence the model by influencing the number of trees and temperature respectively

Saddle-node normal form with steady drift

- One simple system that shows tipping is the saddle-node normal form with steady drift.
- While this is an over-simplified model, it does clearly show a tipping point as a line on the graph.

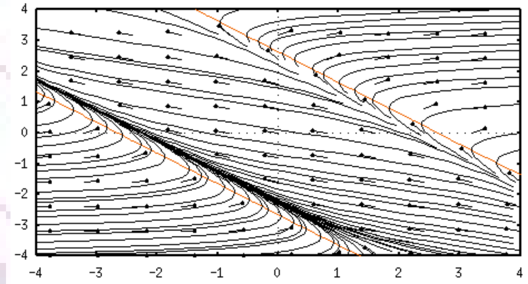
Saddle-node normal form with steady drift

- Equations for the model

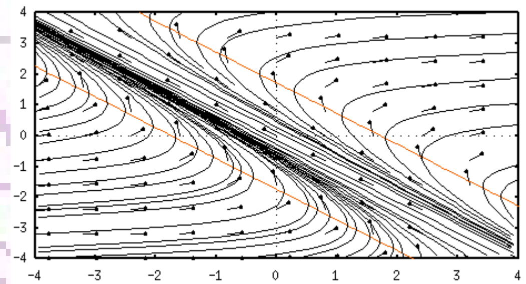
$$\frac{dx}{dt} = (x+\lambda)^2 - \mu,$$
$$\frac{d\lambda}{dt} = r,$$

- Let parameter μ > 0 be fixed, and drift r be fixed.

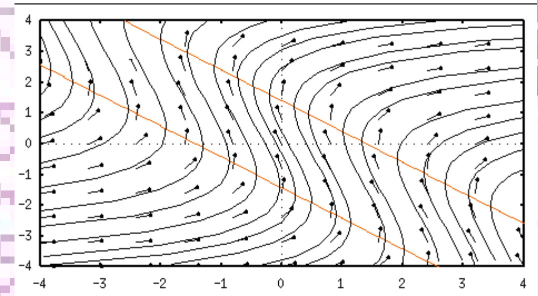
$0 < r < \mu$



$r = \mu$



$r > \mu$



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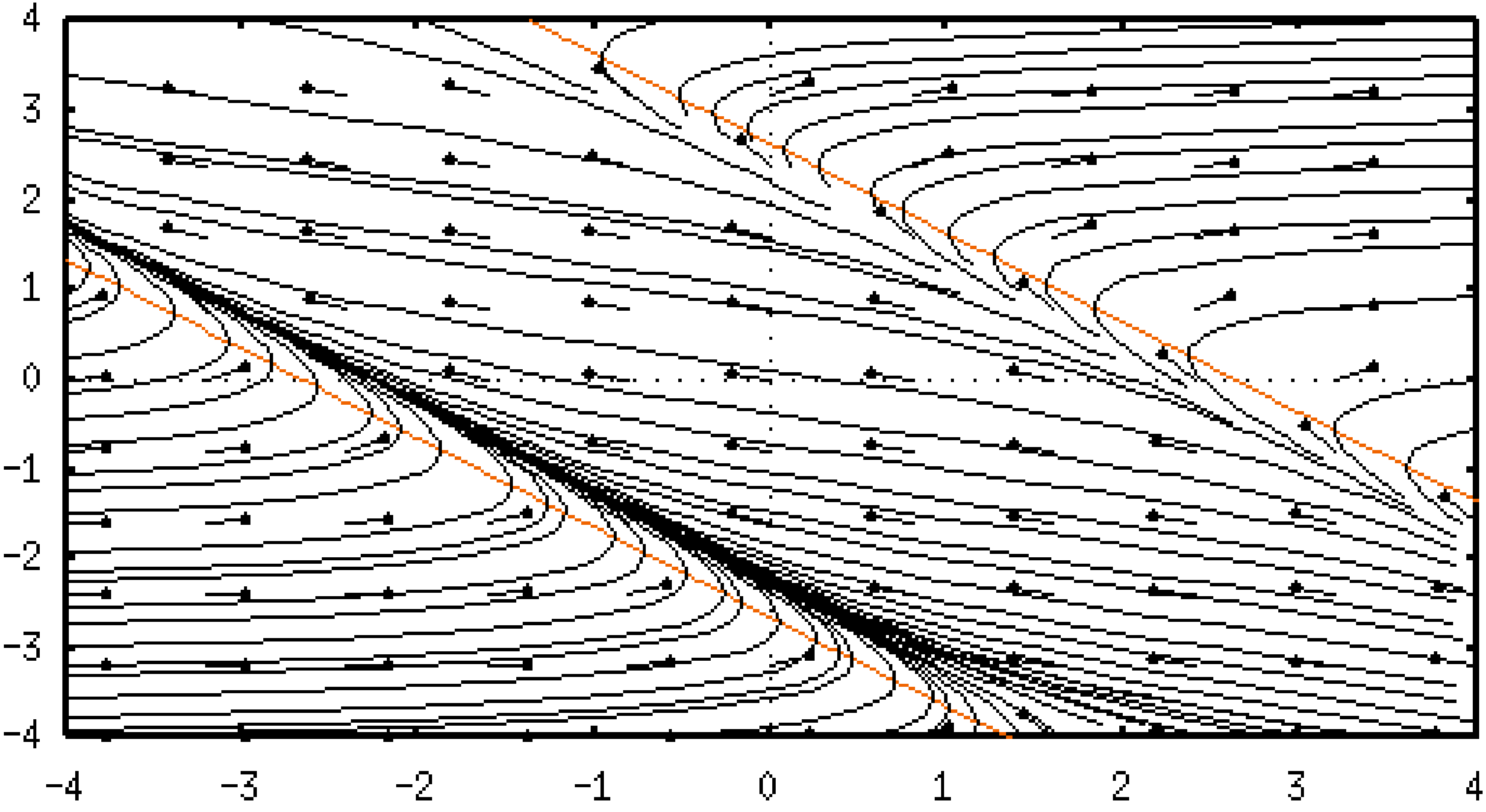

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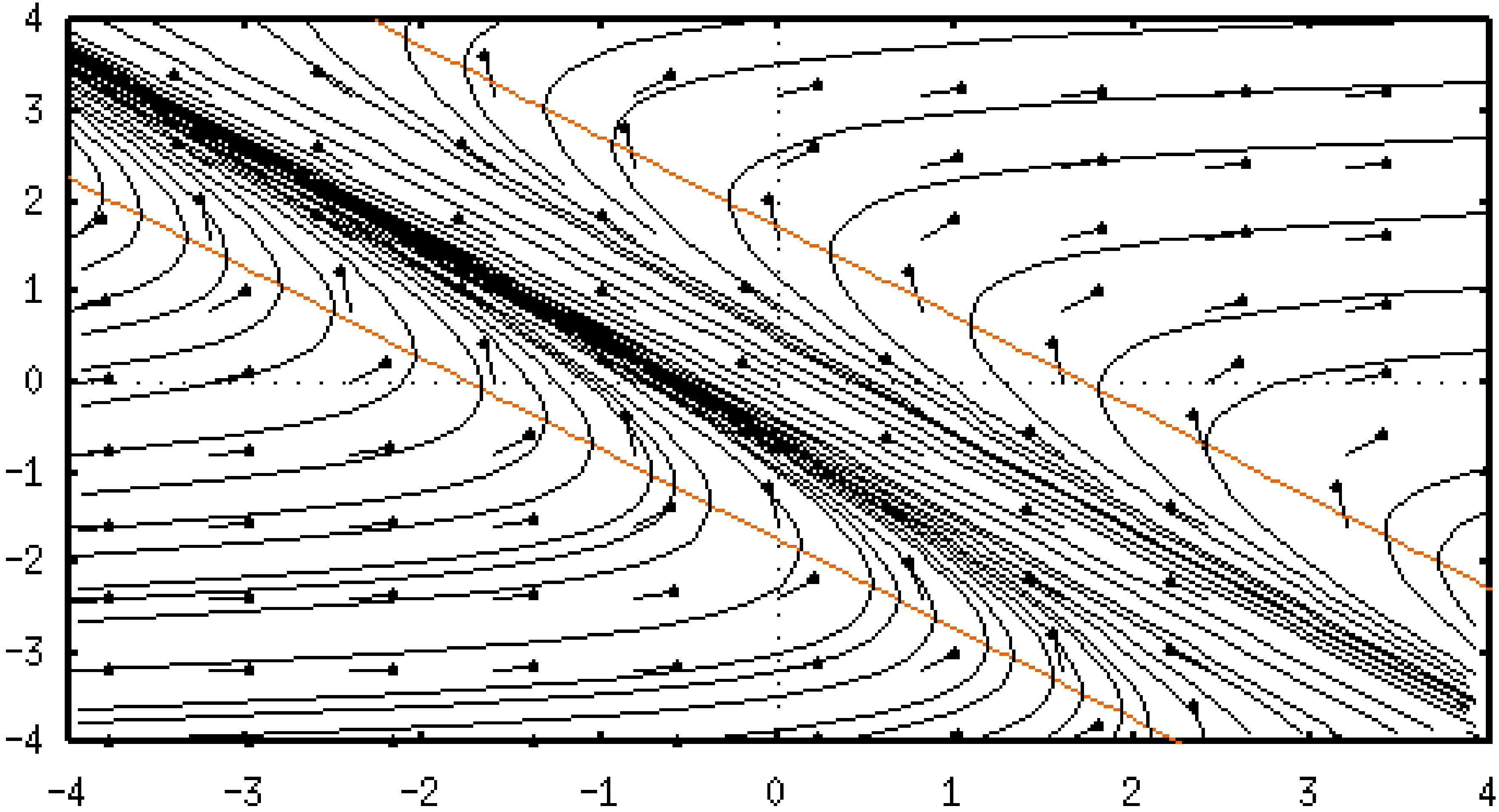

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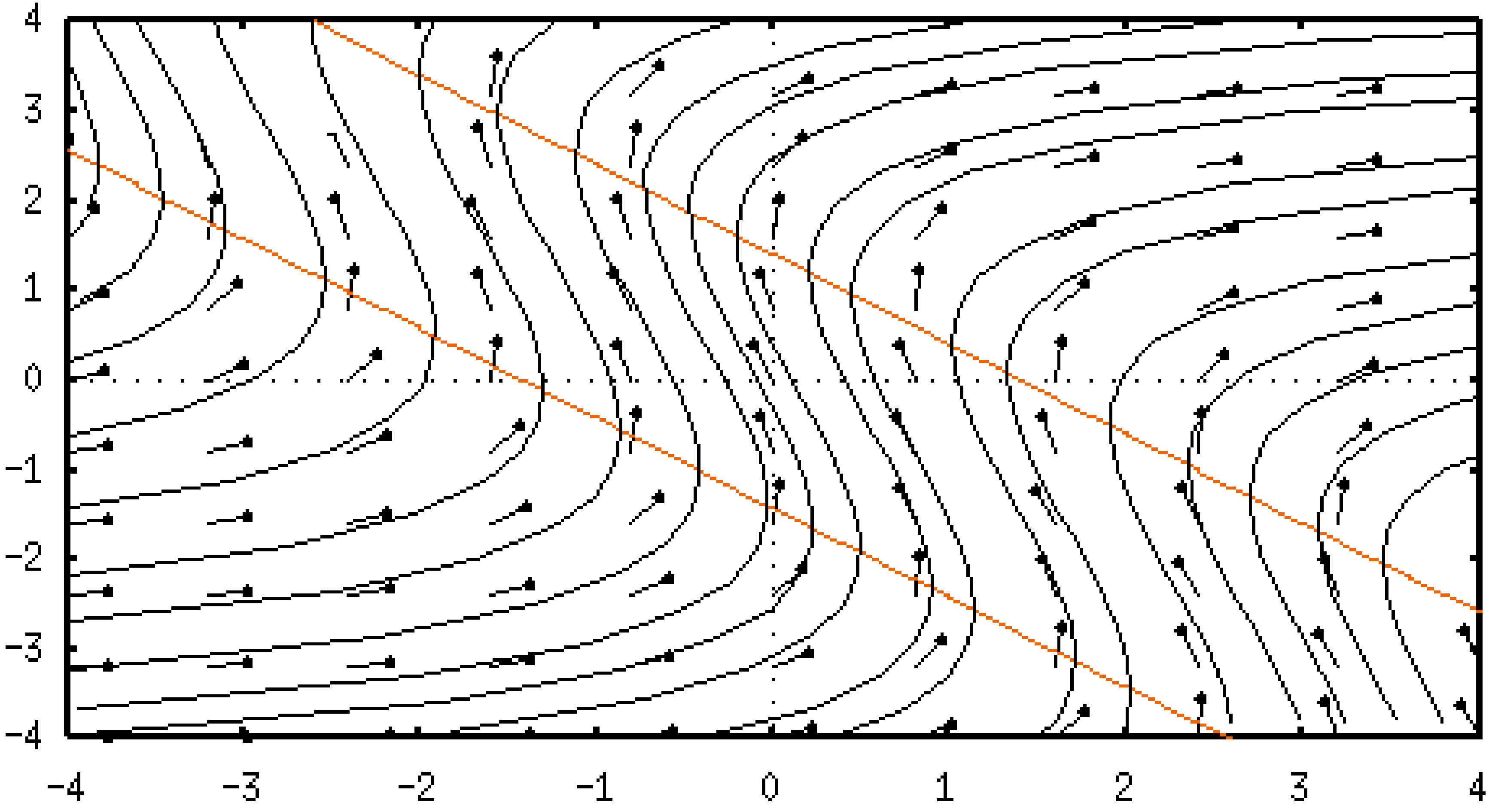
r > μ



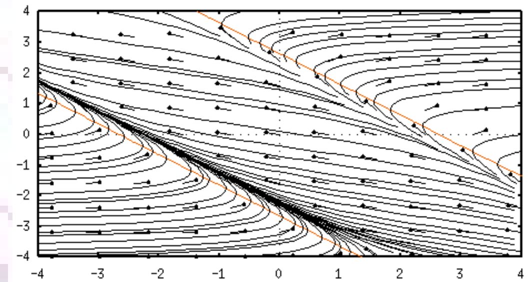
Saddle-node normal form with steady drift

- Equations for the model

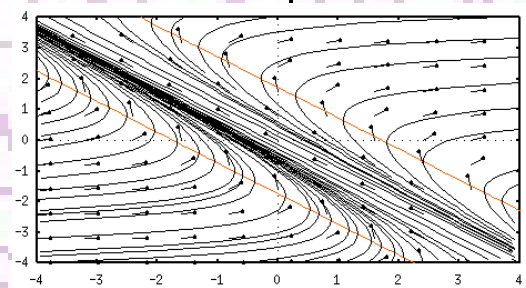
$$\frac{dx}{dt} = (x+\lambda)^2 - \mu,$$
$$\frac{d\lambda}{dt} = r,$$

- Analysis of equations
  - Note the Nullclines at
    - » $\lambda = \sqrt{(\mu)} - x$
    - » $\lambda = -\sqrt{(\mu)} - x$
  - Also, note that there are no fixed points as $\frac{d\lambda}{dt}$ can never equal zero.

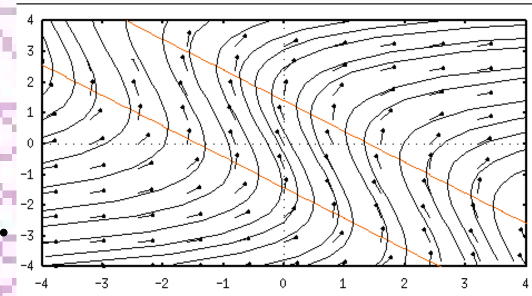
$0 < r < \mu$



$r = \mu$

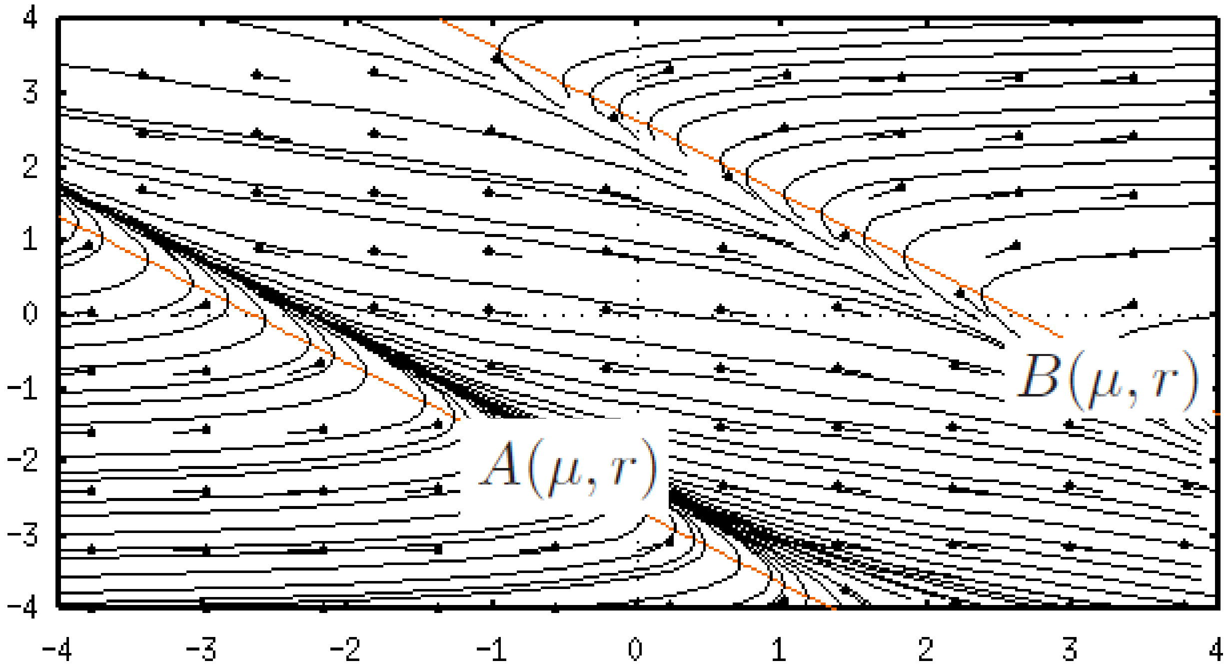


$r > \mu$



Saddle-node normal form with steady drift

$0 < r < \mu$



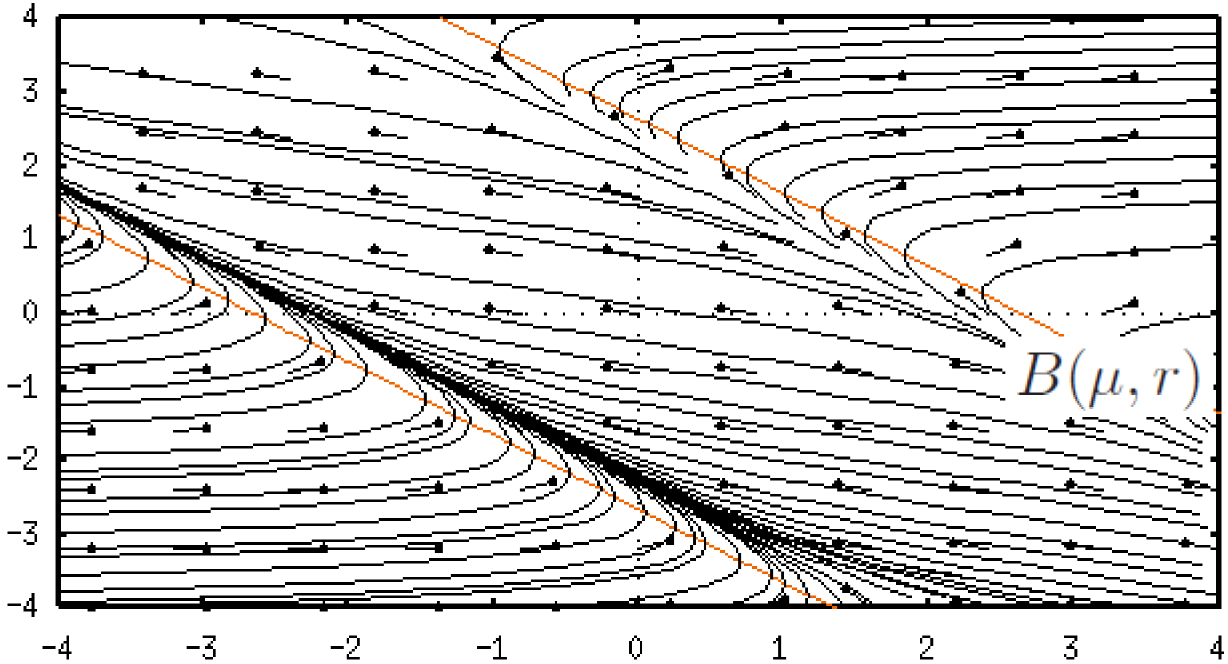
- Note the two invariant lines:

$$A(\mu, r) = \{(x, \lambda) \in \mathbb{R}^2 : \lambda = -\sqrt{\mu - r} - x\}$$

$$B(\mu, r) = \{(x, \lambda) \in \mathbb{R}^2 : \lambda = \sqrt{\mu - r} - x\}$$

Saddle-node normal form with steady drift

$0 < r < \mu$



$$B(\mu, r) = \{(x, \lambda) \in \mathbb{R}^2 : \lambda = \sqrt{\mu - r} - x\}$$

- B(μ, r) defines a tipping *threshold*. Initial conditions below B(μ, r) converge to a parallel line, whereas initial conditions above B(μ, r) have solutions x(t) → ∞ as t → ∞.

Analysis of the Model

- In similar model, we can think of the current climate of forests as *A(μ,r)* and a possible future as *B(μ,r)*. In the present (*A(μ,r)*), we have a moderate temperature and large forests, however if the temperature rises too much, or forests die too much, the climate will change to *B(μ,r)*.

$$\frac{dx}{dt} = -r$$

$$\frac{d\lambda}{dt} = -(x+\lambda)^2 + \mu$$