Always justify your answers, even if the question does not explicitly say so! Write your own solutions, independently of anyone else.

Core Problems: Everyone must turn these problems in.

I. Sec. 6.3 # 10, 20, 21, 26.

II. Find all group homomorphisms \( \phi : \mathbb{Z}_{25} \to \mathbb{Z}_{15} \). Be sure to show that your answers are well-defined homomorphisms, and that you’ve found them all. In each case, find the kernel and the image of \( \phi \).

III. Let \( U_n \) be the set of all \( n^{th} \) roots of unity (\( n \in \mathbb{N} \)). Identify \( S^1 / U_n \) as some familiar group, and prove your answer using the Fundamental Homomorphism Theorem.

IV. Sec. 6.4 # 4.

Advanced Problem: Advanced problems are due one week after the Core problems from the same assignment, unless announced otherwise.

V. Sec. 6.3 # 31.