1. Suppose $u_0, u_1$ are least upper bounds of a set $X \subset \mathbb{R}$. Prove that $u_0 = u_1$.

2. Suppose $u$ is an upper bound of a nonempty set $X \subset \mathbb{R}$. Prove that $u = \sup X$ if and only if for all $\varepsilon > 0$ there exists $x \in X$ with $x > u - \varepsilon$.

3. Define “greatest lower bound.” Prove that every nonempty set $X \subset \mathbb{R}$ which has a lower bound must have a greatest lower bound. (You should use the LUB Property of $\mathbb{R}$; do not try to re-write the proof with all the inequalities reversed!)

4. Find $\sup X$ and $\inf X$ if $X = \left\{ \frac{1}{n} \mid n \in \mathbb{N} \right\}$.

5. Find a discontinuous function $f : [0, 1] \to \mathbb{R}$ which does not satisfy the conclusion of the Intermediate Value Theorem. (The domain of $f$ must not omit any points of $[0, 1]$.)

6. Prove that every continuous function on a closed interval $[a, b]$ attains a minimum value. (You should use the corresponding result for maximum values; do not try to re-write the proof with all the inequalities reversed!)

7. Give examples of the following:

   (a) a continuous function $f : (0, 1) \to \mathbb{R}$ which attains a maximum value but not a minimum value.

   (b) a continuous function $g : (0, 1] \to \mathbb{R}$ which attains neither a maximum nor a minimum value.