Proof: Induct on r. If $r = 2$, the result is obvious. Now if the statement is true for K_r-free graphs it must be shown that K_{r+1}-free graphs have no more than $(r-1)n^2/2r$ edges. Let G be such a graph, and let x be the number of vertices in a largest K_r-free induced subgraph of G. Since the neighbors of any vertex induce a K_r-free subgraph, no vertex of G has degree exceeding x. Let A be a largest induced K_r-free subgraph of G. By induction, there are at most $(r-2)x^2/(2r-2)$ edges in A. Each edge of G not in A is incident with at least one of the $n-x$ vertices not in A, so summing the degrees of these vertices counts each such edge at least once. Hence there are at most $x(n-x)$ such edges and so G has at most $(r-2)x^2/(2r-2) + x(n-x)$ edges. Since

$$\frac{r-2}{2r-2}(x^2) + x(n-x) = \frac{r-1}{2r}n^2 - \frac{r}{2r-2}\left(x - \frac{(r-1)n}{r}\right)^2,$$

the result follows. ∎

Turán's theorem continues, in every graph theory textbook, to be the centerpiece of the presentation of extremal graph theory. For this reason, we hope our short proof will be found worthwhile.

REFERENCE

1. P. Turán, On an Extremal Problem in Graph Theory, *Matematicko Fizicki Lapok* **48** (1941) 436–452.

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A Characterization of the Set of Points of Continuity of a Real Function

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In this note, we prove the converse of the following well known result: the set of points of continuity of an arbitrary real valued function on a metric space is a countable intersection of open sets [**1**, p. 58].

Lemma. *If X is a nonempty metric space without isolated points, then X has a dense subset A whose complement is also dense in X.*

Proof: Call a set $S \subset X$ an ϵ-net if (a) $d(x, y) \geq \epsilon$ for any two distinct points x, y of S, and (b) S is maximal with respect to (a). Zorn's Lemma yields that ϵ-nets exist for every $\epsilon > 0$. Suppose we have disjoint sets $S_1, S_2, \dots, S_k$, where each S_i is an $(1/i)$-net. The complement of $S_1 \cup \cdots \cup S_k$ is then nonempty and has no isolated points, and therefore there is an S_{k+1}, disjoint from $S_1 \cup \cdots \cup S_k$, which is an $(1/(k+1))$-net. Then $A = \bigcup_{n=1}^{\infty} S_{2n}$ and $B = \bigcup_{n=1}^{\infty} S_{2n-1}$ are disjoint, and both are dense in X.

Theorem. *Let X be a nonempty metric space without isolated points. If G is a countable intersection of open sets, then there is a function $\phi(x)$ which is continuous exactly on G.*

Proof: Since G is a countable intersection of open sets, we can denote the complement of G as a union of increasing sequence of closed sets F_n, $n = 1, 2, \ldots$. If we define a function $g: X \to R$ by $g(x) = \sum_{n \in K}(\frac{1}{2})^n$, where $K = \{n : x \in F_n\}$, then $g(x)$ converges to 0 as x goes to a point of G. Choose a subset A of X such that A and A^c are both dense, and let $\phi(x) = g(x)(\chi_A(x) - 1/2)$, where $\chi_A(x)$ is a characteristic function on A. Then every neighborhood of an interior point x of G^c contains a point at which the sign of ϕ is different from the sign of $\phi(x)$. If $x \in \partial G \cap G^c$, then $\phi(x) \neq 0$ and every neighborhood of x contains a point y such that $\phi(y) = 0$. Thus ϕ is not continuous at any point of G^c. On the other hand ϕ is continuous at all points of G since $\phi(x) = 0$ for all $x \in G$, and $\phi(x)$ converges to 0 as x goes to a point of G. Therefore, ϕ is continuous exactly on G. ∎

Since a function is continuous at any isolated point, we obtain the following.

Corollary. *Let X be a nonempty metric space. If G is an intersection of countable open subsets and contains all of the isolated points of X, then there is a real valued function on X which is continuous exactly on G.*

REFERENCE

1. W. Rudin, *Real and Complex Analysis*, second edition, McGraw-Hill, New York, 1974.

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Snow and Ice and Geometry

[my name] is merely a sound. If you look beyond the sound, you will find the body with its circulation, its movement of fluids. Its love of ice, its anger, its longing, its knowledge about space, its weakness, faithlessness and loyalty. Behind these emotions the unnamed forces rise and fade away, parceled-out and disconnected images of memory, nameless sounds. And geometry. Deep inside us is geometry. My teachers at the university asked us over and over what the reality of geometric concepts was. They asked: Where can you find a perfect circle, true symmetry, an absolute parallel when they can't be constructed in this imperfect, external world?

I never answered them, because they wouldn't have understood how self-evident my reply was, or the enormity of its consequences. Geometry exists as an innate phenomenon in our consciousness. In the external world a perfectly formed snow crystal would never exist. But in our consciousness lies the glittering and flawless knowledge of perfect ice.

If you have strength left, you can look further, beyond geometry, deep into the tunnels of light and darkness that exist within each of us, stretching back toward infinity.

There's so much you could do if you had the strength.

Smilla's Sense of Snow, by Peter Høeg, translated by Tiina Nunnally
Dell Publishing, New York, 1994, pp. 318–319

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