CORRIGENDUM: THE MODULAR CURVE X_0 (169) AND RATIONAL ISOGENY

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The error arose from equation (2) (in [**2**]) which should have read

$$XY\{X^{12}+X^{11}Y+\ldots+Y^{12}+\ldots+124852(X+Y)+15145\}-13 = 0. \qquad (2)$$

Consequently, the only possible values of X and Y modulo 3, rational over $\mathbb{F}_3$, are $X \equiv \pm 1(3)$ and $X \equiv Y(3)$. Since $W = W_{169}$ permutes X and Y, these two points are fixed by W. The modular invariant corresponding to them is the supersingular invariant $j = 0$ in characteristic 3.

As in [**1**], we calculated the characteristic polynomial of the Hecke operator T_p for $p = 2$ and 3. The polynomial for T_2 is $(X^3-2X^2-X+1)(X^3+2X^2-X-1)(X^2-3)$, and that for T_3 is $(X^3+2X^2-X-1)^2(X-2)^2$. From these it is clear that the Eisenstein quotient $J^{(7)} = \tilde{J}_0(169)$ has Mordell–Weil group of order 7 over $\mathbb{Q}$ which is generated by the image of the class of the divisor P_0-P_∞. Also the reduction homomorphism on $J^{(7)}(\mathbb{Q})$ is injective modulo 2, 3 and 13.

Let $P \in Y_0(169)(\mathbb{Q})$. Since the elliptic curve in a rational pair (E, A) corresponding to P has potentially good reduction modulo 3, the image of the divisor class $P-WP$ on $J^{(7)}$ reduces to 0 modulo 3. Hence P cannot reduce to $P_0/\mathbb{F}_2$ or $P_\infty/\mathbb{F}_2$ modulo 2, or to $P_0/\mathbb{F}_{13}$ or $P_\infty/\mathbb{F}_{13}$ modulo 13.

Consequently, the only possibilities for X and Y are

(i) $X = Y = \varepsilon 13^r/m$,

(ii) $X = 13Y = \varepsilon 13/m$,

where $\varepsilon = \pm 1$, r is an integer and m is a positive integer divisible only by primes p congruent to 1 modulo 13. Both cases are easily dismissed.

References

1. M. A. Kenku, "The modular curves $X_0(65)$ and $X_0(91)$ and rational isogeny", *Math. Proc. Cambridge Philos. Soc.*, 87 (1980), 15–20.
2. M. A. Kenku, "The modular curve $X_0(169)$ and rational isogeny", *J. London Math. Soc.* (2), 22 (1980), 239–244.

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