MATH 3200 PRACTICE PROBLEMS 1 (2016 EDITION)

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In all of the following questions, let x, y, z be objects and A, B, C be sets.

1) Let A and B be sets.
a) What is the meaning of $A = B$? Of $A \subset B$? Of $A \subsetneq B$?
b) What is the meaning of $A \cup B$? Of $A \cap B$? Of $A \setminus B$?

2) Define the **symmetric difference** of A and B as $(A \setminus B) \cup (B \setminus A)$.
In this problem, the symmetric difference will be denoted as $A \Delta B$.
a) Draw a Venn diagram indicating the symmetric difference.
b) Let $A = \{2k \mid k \in \mathbb{Z}^+\}$ and let B be the set of prime numbers. What is $A \Delta B$?
c) Show that $(A \Delta B) \cup (A \cap B) = A \cup B$.

3) a) What is meant by a universal set?
b) What is meant by $\overline{A}$, and when is it defined?

4) a) What is a partition of a set S?
b) Are there any sets S which have no partitions? Which sets S have exactly one partition?

4) Decide whether each of the following is true or false, and briefly explain.
a) Any partition of a finite set must have a finite number of elements.
b) Any partition of an infinite set must have an infinite number of elements.

5) a) Define: implication, contrapositive, converse, inverse, biconditional.
b) What is meant by the "converse fallacy"?
c) Show that $A \implies B$ is logically equivalent to $\neg B \implies \neg A$ in two different ways: (i) using a truth table, and (ii) using an argument in plain English.

6) True or false: Neither the inverse nor the converse of an implication is logically equivalent to the implication, but the inverse and the converse are logically equivalent to each other.

7) Negate the following sentence: "You can fool some of the people all of the time, and all of the people some of the time, but you cannot fool all of the people all of the time."

8) What does it mean for an implication to hold trivially? To hold vacuously?

10) For which of the following famous theorems does the converse also hold?[1]

[1] Moral: when you see a theorem for the first time, ask yourself whether the converse is true!

a) (Pythagorean Theorem) Let a, b and c be the side lengths of a triangle T. If T is a right triangle and c is the length of the hypotenuse, then $c^2 = a^2 + b^2$.
b) (Rational Roots Theorem) Let $P(x) = a_n x^n + \ldots + a_1 x + a_0$ be a polynomial with integer coefficients $a_0, \ldots, a_n$, and let $r = \frac{c}{d}$ be a rational number written in lowest terms. If $P(r) = 0$, then $d \mid a_n$ and $c \mid a_0$.

11) True or false: it is not possible to prove a theorem by giving a single example. Discuss.

12) Let $y \in \mathbb{Z}$ and let $x = 6y + 3$. Show that $3 \mid x$ and $2 \nmid x$.

13) Let $x \in \mathbb{Z}$. Show that if $7 \mid x^2 + 1$, then $13 \mid x^3 + 5x^2 + 17x - 100$.

14) Suppose that $3x + 2$ is odd. Show that $x^4 + 2x + 2009$ is even.