MATH 3200 SECOND MIDTERM EXAM

Directions: Do all five problems. Always justify your reasoning completely. No calculators are permitted (nor would they be helpful in any way that I can see).

1) Let N_0 be an integer, and let $P(n)$ be an open sentence whose domain is the set of integers $n \geq N_0$. State the version of the principle of mathematical induction that allows you to conclude that $P(n)$ holds for all $n \geq N_0$.

2) a) Let $x \in \mathbb{Z}$. Show that x^2 is congruent to 0, 1 or 4 modulo 5.
b) Suppose that $x, y, z \in \mathbb{Z}$ are such that $x^2 + y^2 = z^2$. Show that $5 \mid xyz$.

3) Suppose that x is odd. Show that $x^2 \equiv 1 \pmod{8}$.

4) Show that for all positive integers n, $\sum_{k=1}^{n} \frac{1}{k^2+k} = \frac{n}{n+1}$.

5) Show that there do not exist integers $x, y \in \mathbb{Z}$ such that $x^2 - y^2 = 2$.