**Math 3100** **Spring 2018**

# Sample Exam 1 – Version 3

*No calculators. Show your work. Give full explanations. Good luck!*

1. (15 points) Evaluate the following limits or explain why it is divergent.

   (a) $\lim_{n\to\infty}\left(\frac{2n+1}{3-n}\right)^3$

   (b) $\lim_{n\to\infty}\left((-1)^n+\frac{1}{n}\right)$

   (c) $\lim_{n\to\infty}\frac{\cos(n)}{n^2}$

   (d) $\lim_{n\to\infty}\frac{n!+n}{2^n+3n!}$

   (e) $\lim_{n\to\infty}\frac{n+\log(n)}{n+1}$

2. (20 points)

   (a) Carefully state the definition of the convergence of a sequence $\{a_n\}$ to a real number $L$.

   (b) Using the definition of convergence, prove that

   $$\lim_{n\to\infty}\frac{5n+4}{2n-7}=5/2.$$

3. (15 points)

   (a) Carefully state the definition of a sequence $\{a_n\}$ being bounded above.

   (b) Use this definition to prove that the sequence $a_n=\sqrt{n}$ is <u>not</u> bounded above.

4. (20 points) Assume that $\lim_{n\to\infty} b_n = L > 0$. Using <u>only</u> the definition of convergence prove the following two statements;

   (a) There exists $M>0$ such that $|b_n|\le M$ for all $n\in\mathbb{N}$.

   (b) There exists some $N\in\mathbb{N}$ such that if $n>N$, then $b_n>L/2$.

5. (15 points) Prove that if $\{a_n\}$ is an increasing sequence with $\lim_{n\to\infty} a_n = L$, then $a_n\le L$ for all $n\in\mathbb{N}$.

6. **(Bonus points)** Let $\{a_n\}$ be the Fibonacci sequence given recursively by $a_1=1$ $a_2=1$ and $a_{n+1}=a_n+a_{n-1}$ for $n>1$. We now construct a new sequence $\{b_n\}$ by setting $b_n=a_{n+1}/a_n$ for all $n\in\mathbb{N}$.

   (a) Show that $\{b_n\}$ satisfies the recursive formula $b_{n+1}=1+1/b_n$ with $b_1=1$, and that $1\le b_n\le 2$ for all $n\in\mathbb{N}$.

   (b) It follows from the formula in (a) that $b_{n+2}=1+\frac{b_n}{1+b_n}$ for all $n\in\mathbb{N}$, using this and induction prove that

      i. the subsequence $\{b_{2n}\}$ is decreasing

      ii. the subsequence $\{b_{2n-1}\}$ is increasing

   (c) Conclude that $\{b_n\}$ converges and $\lim_{n\to\infty} b_n = \frac{1+\sqrt{5}}{2}$