Math 3100 Assignment 7
Power Series and Continuity

Due at 12:00 pm on Friday the 9th of March 2018

1. Find a power series representation for the function and determine the interval of convergence.

   (a) $f(x) = \dfrac{1}{1+x}$ (b) $g(x) = \dfrac{1}{1-4x^2}$ (c) $h(x) = \dfrac{1}{4+x^2}$ (d) $F(x) = \dfrac{x}{x-3}$

2. Find all $x \in \mathbb{R}$ for which the following power series converge:

   (a) $\displaystyle\sum_{n=0}^{\infty} n^3 x^n$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!} x^n$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^2} x^n$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{3^n} x^n$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{\sqrt{n}}$

3. Find the *radius of convergence* and *interval of convergence* of the power series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n+3}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n 2^n}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{(n+1)^2}$ (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{\sqrt{n}}$

4. Prove that each of the following functions are continuous at x_0 using the ε-δ definition of continuity.

   (a) $f(x) = 3x^2$, $x_0 = 2$
   (b) $g(x) = \dfrac{2x-3}{x-1}$, $x_0 = 2$
   (c) $h(x) = \dfrac{x^2-x+3}{x+1}$, $x_0 = 1$
   (d) $F(x) = x^3$, x_0 arbitrary
   (e) $G(x) = \dfrac{1}{x^2}$, $x_0 \neq 0$ arbitrary

5. Define a *modified Dirichlet's function* $h : \mathbb{R} \to \mathbb{R}$, by

$$h(x) := \begin{cases} x & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}.$$

   Prove that h is continuous at $x = 0$, but discontinuous at all $x \neq 0$.