Math 3100 Assignment 1

Preliminaries

Due at the beginning of class on Wednesday the 17th of January 2018

- 1. (Induction)
 - (a) Prove, by induction, that the following identities hold for all $n \in \mathbb{N}$:

i.
$$1+2+\dots+n = \frac{n(n+1)}{2}$$

ii. $1^3+2^3+\dots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

It seems like the two identities above must be closely related to each other. Challenge: Can you give a geometric proof of the second identity using only the first?

- (b) Prove, by induction, that the following inequalities hold for all $n \in \mathbb{N}$:
 - i. $2n+1 \le 3n^2$
 - ii. $2n^2 1 \le n^3$

Hint: The validity of the first inequality should help you establish the second.

- 2. (Absolute Value and Inequalities)
 - (a) i. If |x| < 2, what can you say about |x − 3|?
 ii. If |x − 2| < 1, what can you say about |x + 3|?
 iii. If |x + 1| < 1/2, what can you say about |x|⁻¹?
 - (b) Use the triangle inequality to show that

$$\left||x| - |y|\right| \le |x - y|$$

for all $x, y \in \mathbb{R}$. This inequality is often vefered to as the reverse triangle inequality.