Math 3100 Assignment 9

Taylor Series

Homework due date: 1:00 pm on Friday the 9th of November 2018

1. Find a power series representation for the function

   (a) $f(x) = \dfrac{1}{4+x^2}$ (b) $g(x) = \dfrac{1}{(1+x)^2}$ (c) $h(x) = x\log(1+x)$

2. Evaluate these sums

   (a) $\displaystyle\sum_{n=0}^{\infty} 2^{-n}$ (b) $\displaystyle\sum_{n=3}^{\infty} \frac{4^{1-n}}{2n-1}$ (c) $\displaystyle\sum_{n=1}^{\infty} n^2 3^{-n}$

3. Find the Taylor Polynomial of order n generated by f centered at x_0.

   (a) $f(x) = \log x, \quad x_0 = 1, \quad n = 3$
   (b) $f(x) = \sqrt{x+4}, \quad x_0 = 0, \quad n = 2$
   (c) $f(x) = \dfrac{xe^{-x}}{x^2+1}, \quad x_0 = 0, \quad n = 6$

4. Let $f(x) = \dfrac{1}{1+3x^2}$. Without differentiating, find $f^{(8)}(0)$. Show your work.

5. Find the Taylor Series centered at $x_0 = 0$ (the Maclaurin Series) of the following functions.

   (a) $x^2 \sin x$
   (b) $\sin^2 x$ *Hint:* $\sin^2 x = (1 - \cos 2x)/2$.

6. Find the Taylor series generated by f at x_0.

   (a) $f(x) = x^4 + x^2 + 1, \quad x_0 = -2$
   (b) $f(x) = x^{-2}, \quad x_0 = 1$

7. For what values of x do the following polynomials approximate $\sin x$ to within 0.01

   (a) $P_1(x) = x$ (b) $P_3(x) = x - x^3/6$ (c) $P_5(x) = x - x^3/6 + x^5/120$

8. How accurately does $1 + x + x^2/2$ approximate e^x for $-1 \le x \le 1$? Can you find a polynomial that approximates e^x to within 0.01 on this interval?

9. (a) How accurately does $1 - x^2 + x^4/2$ approximate e^{-x^2} for $-1 \le x \le 1$?
   (b) Can you find a polynomial that approximates e^{-x^2} to within 0.01 on this interval?

10. Find a polynomial that will approximate

$$F(x) = \int_0^x t^2 e^{-t^2}\, dt$$

for all x in the interval $[0, 1]$ with an error of magnitude less than 10^{-3}.