Math 2500 **Spring 2015**

Exam 2 - Practice Questions

1. Calculate the iterated integrals.

   (a) $\displaystyle\int_{-2}^{2}\int_{0}^{4}(4x^3+3xy^2)\,dx\,dy$ (b) $\displaystyle\int_{0}^{\pi}\int_{0}^{1}x\cos(xy)\,dy\,dx$

   (c) $\displaystyle\int_{0}^{1}\int_{0}^{y^2}e^{y^3}\,dx\,dy$ (d) $\displaystyle\int_{0}^{1}\int_{0}^{x^2}\int_{0}^{y}y^2z\,dz\,dy\,dx$

2. Calculate the iterated integral by first reversing the order of integration.

   (a) $\displaystyle\int_{0}^{1}\int_{x}^{1}e^{x/y}\,dy\,dx$ (b) $\displaystyle\int_{0}^{1}\int_{y^2}^{1}y\sin(x^2)\,dx\,dy$

3. Calculate the value of the multiple integral.

   (a) $\displaystyle\iint_R \frac{1}{(x-y)^2}\,dA$, where $R=\{(x,y)\,:\,0\le x\le 1,\,2\le y\le 4\}$
   (b) $\displaystyle\iint_R xy\,dA$, where R is bounded by $y^2=x^3$ and $y=x$
   (c) $\displaystyle\iint_R (x^2+y^2)^{3/2}\,dA$, where R is the region in the first quadrant bounded by the lines $y=0$ and $y=\sqrt{3}x$ and the circle $x^2+y^2=9$
   (d) $\displaystyle\iiint_D x^2z\,dV$, where $D=\{(x,y,z)\,:\,0\le x\le 2,\,0\le y\le 2x,\,0\le z\le x\}$
   (e) $\displaystyle\iiint_D yz\,dV$, where D lies above the plane $z=0$, below the plane $z=y$, and inside the cyclinder $x^2+y^2=4$
   (f) $\displaystyle\iiint_D z^3\sqrt{x^2+y^2+z^2}\,dV$, where D is the solid hemisphere with center the origin, radius 1, that lies above the xy-plane

4. Express the integral $\iiint_D dV$ and an iterated integral in six different ways, where D is the solid bounded by the given surfaces.

   (a) $x^2+z^2=4,\ y=0,\ y=6$
   (b) $z=0,\ x=0,\ y=2,\ z=y-2x$
   (c) $z=0,\ z=y,\ x^2=1-y$

5. Find the volume of the given solid.

   (a) Under the paraboloid $z=x^2+4y^2$ and above the rectangle $R=[0,2]\times[1,4]$
   (b) Under the surface $z=x^2y$ and above the triangle in the xy-plane with vertices $(1,0)$, $(2,1)$ and $(4,0)$.
   (c) The solid tetrahedron with vertices $(0,0,0)$, $(0,0,1)$, $(0,2,0)$ and $(2,2,0)$
   (d) Bounded by the cylinder $x^2+y^2=4$ and the planes $z=0$ and $y+z=3$
   (e) Above the paraboloid $z=x^2+y^2$ and below the half-cone $z=\sqrt{x^2+y^2}$

6. (a) Describe the region whose area is given by the following integral and evaluate the integral

$$\int_0^{\pi} \int_1^{1+\sin\theta} r\, dr\, d\theta$$

   (b) Describe the solid whose volume is given by the integral and evaluate the integral

$$\int_0^{2\pi} \int_0^{\pi/6} \int_1^{3} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

7. Use polar coordinates to evaluate

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2}\, dx\, dy$$

8. Use spherical coordinates to evaluate

$$\int_0^{1} \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2+y^2+z^2)^2\, dz\, dy\, dx$$

9. Use cylinderical coordinates to evaluate

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} (x^2+y^2)^{3/2}\, dz\, dy\, dx$$

10. Set up, **but do not evaluate**, the following.

   (a) A double integral in rectangular coordinates for

$$\iint_R xy\, dA$$

   where R is bounded by $y^2 = x^3$ and $y = x$.

   (b) A double integral in polar coordinates for

$$\iint_R (x^2+y^2)^{1/2}\, dA$$

   where R is the region in the first quadrant bounded by the lines $y = 0$ and $y = x/\sqrt{3}$ and the circle $x^2 + y^2 = 9$.

   (c) A triple integral in cylindrical coordinates

$$\iiint_D yz\, dV$$

   where D lies above the plane $z = 0$, below the plane $z = y$, and inside the cyclinder $x^2+y^2 = 4$

   (d) A triple integral in spherical coordinates

$$\iiint_D z^3 \sqrt{x^2+y^2+z^2}\, dV$$

   where D is the solid hemisphere with center the origin, radius 1, that lies above the xy-plane

11. Find the mass and center of mass of a thin plate that occupies the region bounded by the parabola $x = 1 - y^2$ and the coordinate axes in the first quadrant if the density function is $\delta(x, y) = y$.

12. Find the centroid of the solid that is bounded by the parabolic cylinder $z = 1 - y^2$ and the planes $x + z = 1$, $x = 0$ and $z = 0$.

13. Find the centroid of the solid that lies above the cone $\phi = \pi/3$ and below the sphere $\rho = 4\cos\phi$.

14. Find the moment of inertia about a diameter of the base of a solid hemisphere of radius a that has constant density equal to 1.