Practice Exam 1

1. (15 points) Solve the following two differential equations and in each case give a geometric description of the family of all solutions.

   (a) $\dfrac{dy}{dx} = -\dfrac{x}{y}$ (b) $\dfrac{dy}{dx} = \dfrac{y}{x}$, $x > 0$, $y > 0$

2. (10 points) Find the volume of a napkin ring that is constructed by drilling a hole of radius 3 inches through the center of a sphere of radius 5 inches.

3. (20 points) Let $\mathcal{R}$ denote the region in the plane bounded between the curves $y = x^3$ and $x = y^2$.

   (a) Set up, but do not evaluate, an <u>integration in the variable y</u> for the volume obtained by

        i. revolving $\mathcal{R}$ about the x-axis

        ii. revolving $\mathcal{R}$ about the y-axis

   (b) Set up, but do not evaluate, an <u>integration in the variable x</u> for the volume obtained by

        i. revolving $\mathcal{R}$ about the line $y = 1$

        ii. revolving $\mathcal{R}$ about the line $x = 1$

4. (10 points) A solid lies between planes perpendicular to the x-axis at $x = 0$ and $x = 1$. The cross-sections perpendicular to the axis on the interval $0 \le x \le 1$ are squares whose diagonals run from the curve $y = x$ to the curve $y = x^3$. Set up, but do not evaluate, a definite integral for the volume of this solid.

5. (10 points) The tank below is full of water. Set up, but do not evaluate, a definite integral for the work required to empty the tank by pumping the water to a height of 1 meter above the top of the tank.
   (Assume that the water weighs 10,000 N/m^3)

6. (10 points) Find the length of the curve

   $$y = \frac{x^3}{6} + \frac{1}{2x}, \quad 1 \le x \le 2$$

7. (10 points) Find the area of the surface generated by rotating the curve

   $$2y = 2 + x^2, \quad 1 \le x \le 2$$

   about the y-axis.

8. (15 points)

   (a) An empty bucket weighing 5 lb is lifted from the ground into the air by pulling in 20 ft of rope at a constant speed. The rope weighs 0.1 lb/ft. How much work was required to lift the bucket and rope?

   (b) How much work would be required to lift the same bucket and rope if the bucket instead starts by containing 10 lb of water, leaks water at a constant rate, and finishes draining just as it reaches the top?