Math 2260 **Spring 2017**

Exam 3 - Practice Questions

1. Determine whether the sequence converges or diverges. If it converges, find the limit.

   (a) $a_n = \dfrac{n \ln n}{n^2+1}$ (b) $b_n = (1+2n)^{1/n}$ (c) $c_n = \dfrac{\cos\sqrt{n}}{\sqrt{n}}$

2. Test the series for convergence or divergence.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+3}$ (b) $\displaystyle\sum_{n=0}^{\infty} \cos(n)$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{3^{2n-1}}$ (d) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{8n-5}$ (f) $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n(\ln n)^3}$ (g) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$ (h) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{5^n+n}$

3. Test the series for convergence or divergence.

   (a) $\displaystyle\sum_{n=1}^{\infty} \sin(1/n)$ (b) $\displaystyle\sum_{n=1}^{\infty} n\sin(1/n)$ (c) $\displaystyle\sum_{n=1}^{\infty} \ln(1+n^{-2})$

4. Find the *radius of convergence* and *interval of convergence* of the power series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n+3}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n2^n}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{(n+1)^2}$

   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{4^n}(2x-1)^n$ (e) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^n}{\sqrt{n}}$ (f) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^n}$

5. Find a power series representation for the function and determine the interval of convergence.

   (a) $f(x) = \dfrac{1}{4+x^2}$ (b) $g(x) = \dfrac{1}{(1+x)^2}$ (c) $h(x) = x\ln(1+x)$

6. Find the series' *interval of convergence* and, within this interval, the actual sum of the series as a function of x.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^{2n}}{9^n}$ (b) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{x^2-1}{2}\right)^n$ (c) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$

7. Find the Taylor series for the function f at a.

   (a) $\sin 2x, \quad a = 0$
   (b) $1 + x + x^2 + x^3, \quad a = 1$
   (c) $xe^{-x}, \quad a = 0$

8. Find the Taylor polynomial $P_n(x)$ for the function f at a.

   (a) $\cos x, \quad a = \pi/6, \quad n = 3$
   (b) $\tan x, \quad a = 0, \quad n = 4$
   (c) $\sqrt{x}, \quad a = 9, \quad n = 3$

9. For what values of x do the following polynomials approximate $\sin x$ to within 0.01

   (a) $P_1(x) = x$ (b) $P_3(x) = x - x^3/6$ (c) $P_5(x) = x - x^3/6 + x^5/120$

10. How accurately does $1+x+x^2/2$ approximate e^x for $-1 \le x \le 1$? Can you find a polynomial that approximates e^x to within 0.01 on this interval?