Exam 1 - Practice Questions

1. (a) Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = x^2 - 3x + 2$ and $y = 0$.

   (b) Let $\mathcal{R}$ be the region in the first quadrant bounded by the curves $y = x^3$ and $y = 2x - x^2$. Set up an integration in the variable x for the following quantities:

   i. The volume obtained by revolving $\mathcal{R}$ about the x-axis
   ii. The volume obtained by revolving $\mathcal{R}$ about the line $y = 2$
   iii. The volume obtained by revolving $\mathcal{R}$ about the y-axis
   iv. The volume obtained by revolving $\mathcal{R}$ about the line $x = -2$

2. Suppose you make napkin rings by drilling holes with different diameters through wooden balls (which also have different diameters). You discover that both napkin rings have the same height h.

   (a) Guess which ring has more wood in it.

   (b) Check your guess: Use either the "shell" or "washer" method to compute the volume of a napkin ring created by drilling a hole with radius r through the center of a sphere of radius R and express the answer in terms of h.

3. (a) A solid has a circular base of radius 1. Parallel cross-sections perpendicular to the base are squares. Find the volume of the solid.

   (b) The base of a solid is the region bounded by the parabolas $y = x^2$ and $y = 2 - x^2$. Find the volume of the solid if the cross-sections perpendicular to the x-axis are

   i. semi-circles with one side lying along the base.
   ii. isosceles right triangles with hypotenuse in the base.

4. Set up, but do not evaluate, an integral for the following quantities:

   (a) the length of the curve $y = x^3$, $0 \le x \le 1$
   (b) the length of the curve $y = e^x \cos x$, $0 \le x \le \pi/2$
   (c) the area of the surface generated by revolving $x = \sin y$, $0 \le y \le \pi$ about the y-axis

5. (a) i. Find the length of the curve $y = x^3/6 + 1/2x$, $1 \le x \le 2$
      ii. Find the area of the surface generated by rotating the curve in part (i) about the x-axis.

   (b) Find the area of the surface generated by rotating the curve $y = x^3$, $0 \le x \le 2$ about the x-axis.

6. If 6 J of work are needed to stretch a spring from 10 cm to 12 cm and another 10 J are needed to stretch it from 12 cm to 14 cm, what is the natural length of the spring?

7. A uniform cable hanging over the edge of tall building is 40 ft long and weighs 60 lb. How much work is required to pull 10 ft of the cable to the top?

8. A circular swimming pool has a diameter of 24 ft, the sides are 5 ft high, and the depth of the water is 4 ft. How much work is required to pump all of the water out over the side?

9. Find the solution to the following differential equations that satisfy the given initial conditions.

   (a) $xyy' = \ln x$, $y(1) = 2$
   (b) $x = -2yy'\sqrt{x^2 + 1}$, $y(0) = 1$

10. A bacteria culture starts with 4000 bacteria, and the population triples every 30 min.

   (a) Find an expression for the number of bacteria after t hours.
   (b) Find the number of bacteria after 20 min.
   (c) When will the population reach 20,000?