Take-Home Math 2250 Review

Due in class on Friday 9th of January

1. Find the most general antiderivative of
   (a) $f(x) = \sqrt[3]{x^2} - \sqrt{x^3}$
   (b) $g(y) = \sin y - \cos 2y$
   (c) $h(z) = 3/z^2 - 5/z^4$

2. Evaluate the following *indefinite integrals.*
   (a)
   $$\int \sin^2 x \, dx$$
   (b)
   $$\int \frac{x}{\sqrt{x^2+2}} \, dx$$
   (c)
   $$\int \frac{1}{\sqrt{x}(x+1)} \, dx$$

3. Solve the following differential equations subject to the given restraints (initial/boundary conditions). In other words, find $f(x)$ given that...
   (a) $f'(x) = 4x + 3$, $f(0) = -9$
   (b) $f'(x) = 4 - 3(1+x^2)^{-1}$, $f(1) = 0$
   (c) $f''(x) = 3e^x + 5\sin x$, $f(0) = 1$, and $f'(0) = 2$
   (d) $f''(x) = x^{-2}$, $x > 0$, $f(1) = 0$, and $f(2) = 0$

4. **Recall that if f is continuous on $[a, b]$, then f is also integrable on $[a, b]$ and the definition of the definite integral simplifies to**
   $$\int_a^b f(x)\, dx = \lim_{n\to\infty} R_n$$
   where
   $$R_n = \frac{b-a}{n} \sum_{k=1}^{n} f\Big(a + k\frac{b-a}{n}\Big).$$
   (a) Use this formula to evaluate the integral
   $$\int_0^2 (x^3 - 3x)\, dx.$$
   (b) Check your answer to part (a) by instead evaluating the integral using the Fundamental Theorem of Calculus (Part II).

5. Use the *Fundamental Theorem of Calculus (Part I)* find $f'(x)$ when

$$f(x) = \int_1^{x^4} \ln t \, dt.$$

6. Use the *Fundamental Theorem of Calculus (Part II)*, or any other technique of your choice, to evaluate the following *definite integrals.*

   (a)
   $$\int_1^2 \frac{x^2+1}{x} \, dx$$

   (b)
   $$\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

   (c)
   $$\int_0^4 \frac{x}{\sqrt{1+2x}} \, dx$$

   (d)
   $$\int_e^{e^4} \frac{1}{x\sqrt{\ln x}} \, dx$$

   (e)
   $$\int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}} \, dx$$

   (f)
   $$\int_0^{\pi/2} \left(\frac{1}{\sqrt{x}} - \sin 2x \right) \, dx$$

   (g)
   $$\int_0^{\pi/2} 2\cos^3 \theta \sin \theta \, d\theta$$

7. Find the area of the region bounded by the graphs of the given equations.

   (a) $y = x^3 - x^2 - 6x$, $y = 0$
   (b) $y = \cos x$, the x-axis, and the vertical lines $x = 0$ and $x = 3\pi/4$
   (c) $y = x^4$ and $y = 2x - x^2$
   (d) $y^2 = 4x$ and $4x - 3y = 4$

8. * Find the function f and the real number a so that

$$6 + \int_a^x \frac{f(t)}{t^2} \, dt = \ln x.$$

9. * Evaluate

$$\lim_{n\to\infty} \left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \cdots + \frac{1}{\sqrt{n}\sqrt{n+n}} \right).$$