

## Old Exam 2

1. (90 points) Evaluate the following

(a)

$$\int x^2 e^{-x} dx$$

(b)

$$\int x e^{-x^2} dx$$

(c)

$$\int x \ln x dx$$

(d)

$$\int \frac{\cos(\ln x)}{x} dx$$

(e)

$$\int \sqrt{4-x^2} dx$$

(f)

$$\int \tan^3 x \sec x dx$$

(g)

$$\int x \tan^{-1} x dx$$

(h)

$$\int \frac{2x-1}{\sqrt{2x-x^2}} dx$$

(i)

$$\int \frac{x-1}{x^2(x^2+1)} dx$$

2. (10 points) Determine if the following improper integrals are convergent or divergent. Evaluate those which converge.

(a)

$$\int_e^\infty \frac{\ln x}{x} dx$$

(b)

$$\int_0^{1/e} \frac{1}{x(\ln x)^2} dx$$

3. (Bonus points) **Gabriel's Horn** is the surface obtained by revolving the curve  $y = 1/x$ ,  $x \geq 1$ , around the  $x$ -axis.

(a) Show that the area under the curve  $y = 1/x$  when  $x \geq 1$ , and the surface area of *Gabriel's Horn* are both infinite.

(b) Show (paradoxically?) that the volume of the solid of revolution enclosed by Gabriel's horn is finite!