

Exam 2 - Practice Questions

ANSWERS

1. $\int \frac{2x+5}{x-3} dx = 2x + 11 \ln|x-3| + C$ (rational function/substitution)
2. $\int x^2 \sqrt{x-1} dx = \frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$ (substitution)
3. $\int e^{x+e^x} dx = e^{e^x} + C$ (substitution)
4. $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx = 2 \sin \sqrt{x} + C$ (substitution)
5. $\int x^2 \sin 2x dx = -\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x + C$ (integration by parts)
6. $\int \sin^{-1} 2x dx = x \sin^{-1} 2x + \frac{1}{2} \sqrt{1-4x^2} + C$ (integration by parts)
7. $\int \tan^2 x dx = \tan x - x + C$ (convert to secants)
8. $\int \cos^4 2x dx = \frac{3}{8}x + \frac{1}{8} \sin 4x + \frac{1}{64} \sin 8x + C$ (half-angle identity)
9. $\int \frac{1}{x^2 \sqrt{1-x^2}} dx = -\frac{\sqrt{1-x^2}}{x} + C$ (trigonometric substitution)
10. $\int \frac{x^2}{x+1} dx = \frac{1}{2}(x-3)(x+1) + \ln|x+1| + C$ (substitution)
11. $\int \frac{x^2}{(x+1)^3} dx = \ln|x+1| + \frac{2}{x+1} - \frac{1}{2(x+1)^2} + C$ (partial fractions)
12. $\int \frac{x^3}{x^2+1} dx = \frac{1}{2}x^2 - \frac{1}{2} \ln|x^2+1| + C$ (rational function)
13. $\int \frac{x^2}{x^3+1} dx = \frac{1}{3} \ln|x^3+1| + C$ (substitution)
14. $\int \frac{x^2+2}{x^3+1} dx = \ln|x+1| + \frac{2}{\sqrt{3}} \tan^{-1} \frac{2x-1}{\sqrt{3}} + C$ (partial fractions)
15. $\int \frac{e^x}{e^{2x}+1} dx = \tan^{-1}(e^x) + C$ (substitution)
16. $\int \ln x dx = x \ln x - x + C$ (integration by parts)

17. $\int \frac{1+x}{1+x^2} dx = \frac{1}{2} \ln|1+x^2| + \tan^{-1} x + C$ (rational function)
18. $\int \sqrt{x} \ln x dx = \frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2} + C$ (integration by parts)
19. $\int e^{2x} \cos 3x dx = \frac{1}{13} e^{2x} (2 \cos 3x + 3 \sin 3x) + C$ (integration by parts)
20. $\int x \sin x \cos x dx = \frac{1}{8} (\sin 2x - 2x \cos 2x) + C$ (integration by parts)
21. $\int (\ln x)^2 dx = x(\ln x)^2 - 2x \ln x + 2x + C$ (integration by parts)
22. $\int x \sin^3(x^2) dx = \frac{1}{6} \cos^3(x^2) - \frac{1}{2} \cos(x^2) + C$ (substitution/trigonometric integral)
23. $\int \sin^2 x \cos^2 x dx = \frac{1}{8} x - \frac{1}{32} \sin 4x + C$ (half-angle identities)
24. $\int x \tan^{-1} x dx = \frac{1}{2} x^2 \tan^{-1} x + \frac{1}{2} \tan^{-1} x - \frac{1}{2} x + C$ (integration by parts)
25. $\int \frac{x^2}{\sqrt{4x-x^2}} dx = 6 \sin^{-1} \left(\frac{x-2}{2} \right) - 4\sqrt{4x-x^2} - \left(\frac{x-2}{2} \right) \sqrt{4x-x^2} + C$ (trig sub)
26. $\int \frac{1}{\sqrt{x^2+4x+8}} dx = \ln(\sqrt{x^2+4x+8} + x + 2) + C$ (complete square/trig sub)
27. $\int \frac{6x^2+5x-3}{x^3+2x^2-3x} dx = \ln|x| + 3 \ln|x+3| + 2 \ln|x-1| + C$ (partial fractions)
28. $\int \frac{\sin x - \cos x}{\sin x + \cos x} dx = -\ln|\sin x - \cos x| + C$ (substitution)
29. $\int \sin^2 2x \cos^3 2x dx = \frac{1}{6} \sin^3 2x - \frac{1}{10} \sin^5 2x + C$ (trigonometric integral)
30. $\int x^3 \ln x dx = \frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 + C$ (integration by parts)
31. $\int \frac{\sqrt{x-2}}{x+2} dx = 2\sqrt{x-2} - 4 \tan^{-1} \left(\frac{1}{2} \sqrt{x-2} \right) + C$ (substitution)
32. $\int \frac{\sqrt{1+\ln x}}{x \ln x} dx = 2\sqrt{1+\ln x} + \ln \left(\frac{\sqrt{1+\ln x} - 1}{\sqrt{1+\ln x} + 1} \right) + C$ (substitution)
33. $\int \tan^3 x \sec^4 x dx = \frac{1}{6} \tan^6 x + \frac{1}{4} \tan^4 x + C$ (trigonometric integral)
34. $\int \frac{x}{x^2+3x+2} dx = \ln(x+2)^2 - \ln|x+1| + C$ (partial fractions)
35. $\int \frac{x^3+x+1}{x^4+2x^2+4x} dx = \frac{1}{4} \ln|x^4+2x^2+4x| + C$ (substitution)

36. $\int \cos \sqrt{x} dx = 2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + C$ (substitution)
37. $\int e^{3x} \cos 5x dx = \frac{1}{34} e^{3x} (5 \sin 5x + 3 \cos 5x) + C$ (integration by parts)
38. $\int \frac{x}{1-x^2 + \sqrt{1-x^2}} dx = -\ln(\sqrt{1-x^2} + 1) + C$ (substitution)
39. $\int \frac{1 + \cos x}{\sin x} dx = \ln |1 - \cos x| + C$ (trigonometric integral)
40. $\int x^5 e^{-x^3} dx = -\frac{1}{3} e^{-x^2} (x^3 + 1) + C$ (integration by parts)
41. $\int \frac{e^{\tan^{-1} x}}{x^2 + 1} dx = e^{\tan^{-1} x} + C$ (substitution)
42. $\int \frac{1}{x^2 - 9} dx = \frac{1}{6} \ln \left| \frac{x-3}{x+3} \right| + C$ (partial fractions)
43. $\int (x + \sin x)^2 dx = \frac{1}{3} x^3 + \frac{1}{2} x = 2 \sin x - \frac{1}{2} \sin x \cos x - 2x \cos x + C$ (expand)
44. $\int \sin x \cos(\cos x) dx = -\sin(\cos x) + C$ (substitution)
45. $\int \frac{x \ln x}{\sqrt{x^2 - 1}} dx = \sqrt{x^2 - 1} (\ln x - 1) + \tan^{-1} \sqrt{x^2 - 1} + C$
46. $\int \sqrt{1+x-x^2} dx = \frac{5}{8} \sin^{-1} \left(\frac{1}{\sqrt{5}} (2x-1) \right) + \frac{1}{4} (2x-1) \sqrt{1+x-x^2} + C$
47. $\int \sqrt{\frac{1+x}{1-x}} dx = \sin^{-1} x - \sqrt{1-x^2} + C$
48. $\int \frac{1}{\sqrt{x+1} + \sqrt{x}} dx = \frac{2}{3} (x+1)^{3/2} - \frac{2}{3} x^{3/2} + C$
49. $\int \frac{1}{1+2e^x - e^{-x}} dx = \frac{1}{3} \ln \left| \frac{2e^x - 1}{e^x + 1} \right| + C$ (substitution)
50. $\int \frac{x^3 + 1}{x^3 - x^2} dx = x + 2 \ln |x-1| - \ln |x| - \frac{1}{x} + C$ (partial fractions)
51. $\int \frac{\ln(x+1)}{x^2} dx = \ln |x| - \left(1 + \frac{1}{x} \right) \ln(x+1) + C$ (integration by parts)
52. $\int \frac{\sin 2x}{\sqrt{9 - \cos^4 x}} dx = -\sin^{-1} \left(\frac{1}{2} \cos^2 x \right) + C$ (substitution)
53. $\int x \sqrt{2x-25} dx = \frac{1}{10} (2x-25)^{5/2} + \frac{25}{6} (2x-25)^{3/2} + C$ (substitution)
54. $\int \sec^3 x dx = \sec x \tan x + \ln |\sec x + \tan x| + C$ (integration by parts)