

MATH 4780/6780: MATHEMATICAL BIOLOGY

Problem Set 4

The assignment is due **Friday 2/28 by 4pm**. Place your assignment in the mailbox of Nicole Song, located in Boyd 434A. Show your work on all problems. Correct answers without the necessary work will not get any credit. Submit your solutions in order (do not place all codes or figures at the end.)

1. (10 pts.) Exercise 3.9.3 from textbook.
2. (15 pts.) Exercise 3.9.4 from textbook.
3. (15pts.) Exercise 3.9.16(a-d) from textbook. Let $\beta = 0$ except for part (a). Combine (b) and (c) as we did in class.
4. (25pts.) Consider the following predator-prey model ($a > 0$).

$$\begin{aligned}x' &= x^2(1 - x) - xy \\y' &= xy - ay\end{aligned}$$

- (a) (5 pts.) Find all fixed points.
- (b) (10 pts.) Analyze the stability of all fixed points. If the stability type depends on a , identify the ranges of a for each stability type (stable node, unstable node, stable spiral, unstable spiral, saddle, center.)
- (c) (10 pts.) Since the stability of some fixed points may depend on a , phase plane will look different for various values of a . Draw the phase-planes (including the nullclines and the vector field) for each different case.