

MATH 4780/6780: MATHEMATICAL BIOLOGY

Problem Set 1

The assignment is due **Friday 1/24 by 4pm**. Place your assignment in the mailbox of Nicole Song, located in Boyd 434A. Show your work on all problems. Correct answers without the necessary work will not get any credit. Submit your solutions in order (do not place all codes or figures at the end.)

1. (10 pts.) Exercise 2.4.2 from textbook. “Fixed point” and “steady state value” are the same. Please ignore part (d).
2. (10 pts.) Exercise 2.4.5 from textbook. In part (c), “graphical stability analysis” means cobwebs.
3. (10 pts.) Consider the modified discrete logistic equation for the Allee effect:

$$N_{t+1} = rN_t^2(1 - N_t)$$

Use Desmos (link available at the course website) to observe the listed cases below, print a representative cobweb figure for each case, and write down the approximate values for a , b , c , d , e .

- (a) For $0 < r < a$, no stable positive steady-state exists.
 - (b) For $a < r < b$, a single stable positive steady-state exists.
 - (c) For $b < r < c$, a stable 2-cycle exists.
 - (d) For $c < r < d$, a stable 4-cycle exists.
 - (e) For $d < r < e$, a stable 8-cycle exists.
 - (f) For $r > e$, chaotic solution exists.
4. (5 pts.) For the previous problem, find the exact value of a , by observing the fact that $y = f(x) = ax^2(1 - x)$ is tangent to $y = x$. In other words, $f'(x^*) = 1$ and $f(x^*) = x^*$.
 5. (10 pts.) Exercise 2.4.10 from textbook. “Orbit of period 2” and “2-cycle” are the same.