Modifying the SEIR Model for Schistosomiasis Simulation

Bovine Tuberculosis

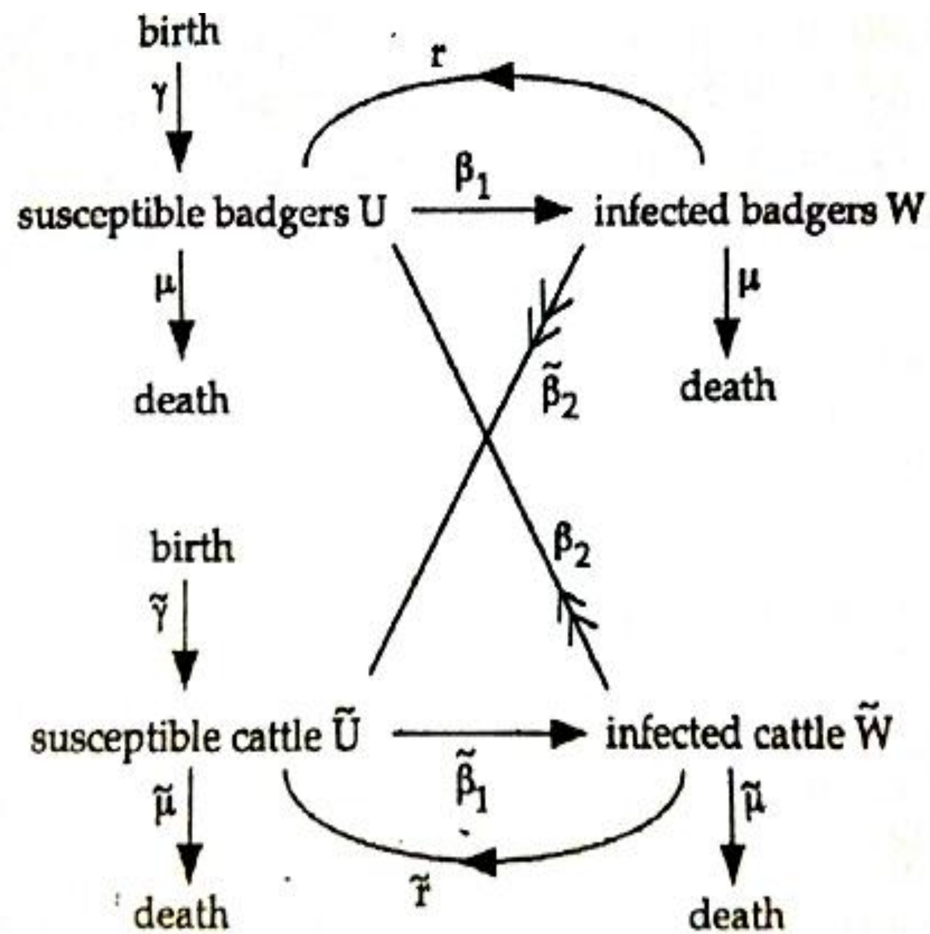
- Based on SEIR model

$$\frac{dU}{dt} + \frac{dU}{da} = -\lambda_1 U + rW - \mu U$$

$$\frac{dW}{dt} + \frac{dW}{da} = \lambda_1 U - rW - \mu W$$

$$\frac{d\tilde{U}}{dt} + \frac{d\tilde{U}}{da} = -\tilde{\lambda}_1 \tilde{U} + \tilde{r}\tilde{W} - \tilde{\mu}\tilde{U}$$

$$\frac{d\tilde{W}}{dt} + \frac{d\tilde{W}}{da} = \tilde{\lambda}_1 \tilde{U} - \tilde{r}\tilde{W} - \tilde{\mu}\tilde{W}$$



Assumptions

- Disease induced death vs. normal death
- Death rates
- Contraction does not result in immunity
- Recovery rate r is proportional to W
- Birth rates and boundary conditions

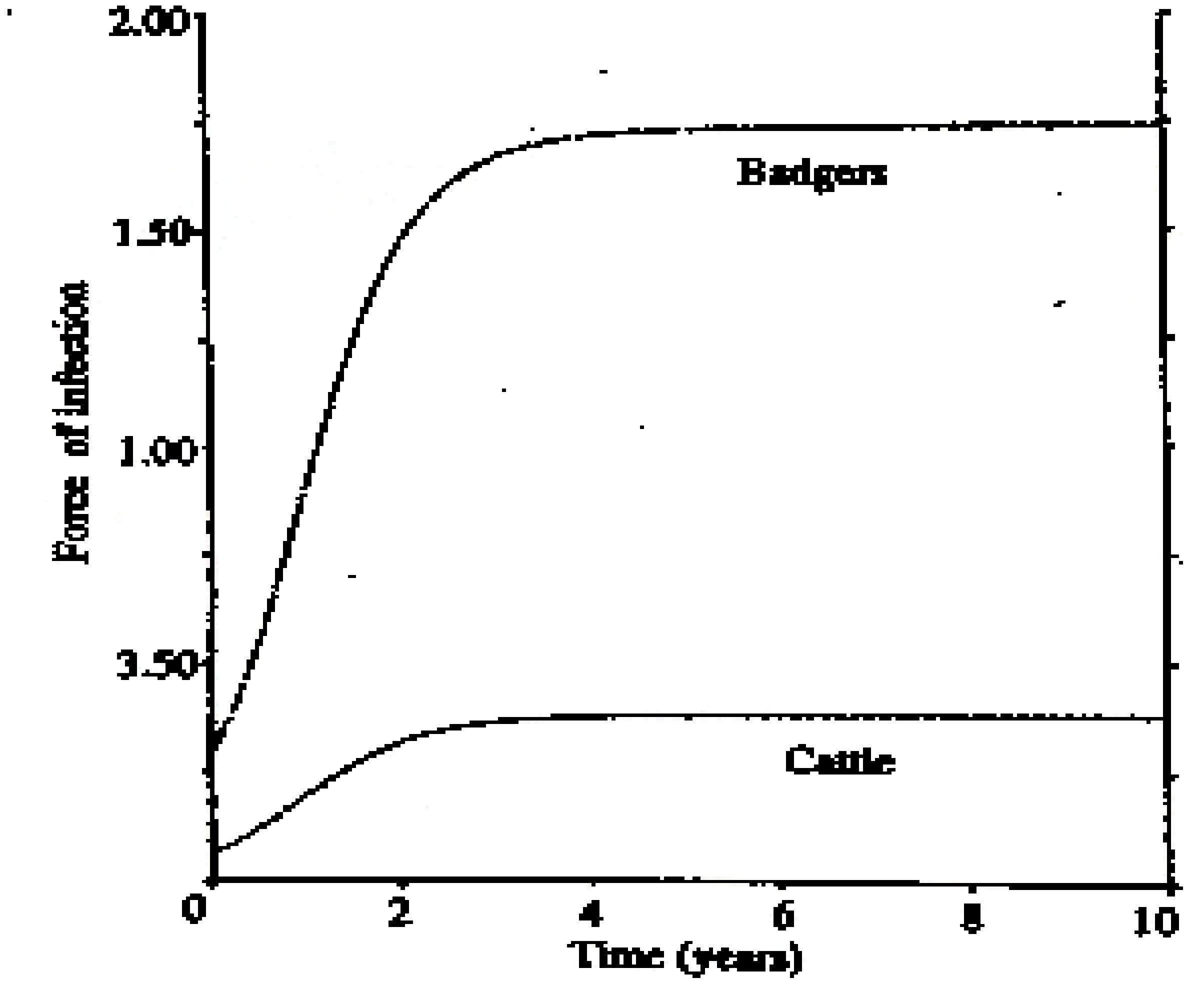
Resulting Equations

$$\lambda_1(t) = \beta_1 \int_0^\infty W(t,a)da + \beta_2 \int_0^\infty \widetilde{W}(t,a)da$$
$$\tilde{\lambda}_1(t) = \tilde{\beta}_1 \int_0^\infty \widetilde{W}(t,a)da + \tilde{\beta}_2 \int_0^\infty W(t,a)da$$

$$\lambda_1(\tau) = \frac{\beta_1}{r} \int_0^\infty w(\tau,\alpha)N(\tau,\alpha)d\alpha + \frac{\beta_2}{r} \int_0^\infty \widetilde{w}(\tau,\alpha)\widetilde{N}(\tau,\alpha)d\alpha$$
$$\tilde{\lambda}_1(\tau) = \frac{\tilde{\beta}_1}{r} \int_0^\infty \widetilde{w}(\tau,\alpha)\widetilde{N}(\tau,\alpha)d\alpha + \frac{\tilde{\beta}_2}{r} \int_0^\infty w(\tau,\alpha)N(\tau,\alpha)d\alpha$$

- Force of infection
- Non-dimensionalising
- Evaluating the λ's to study spread

Simulation Results



Schistosomiasis

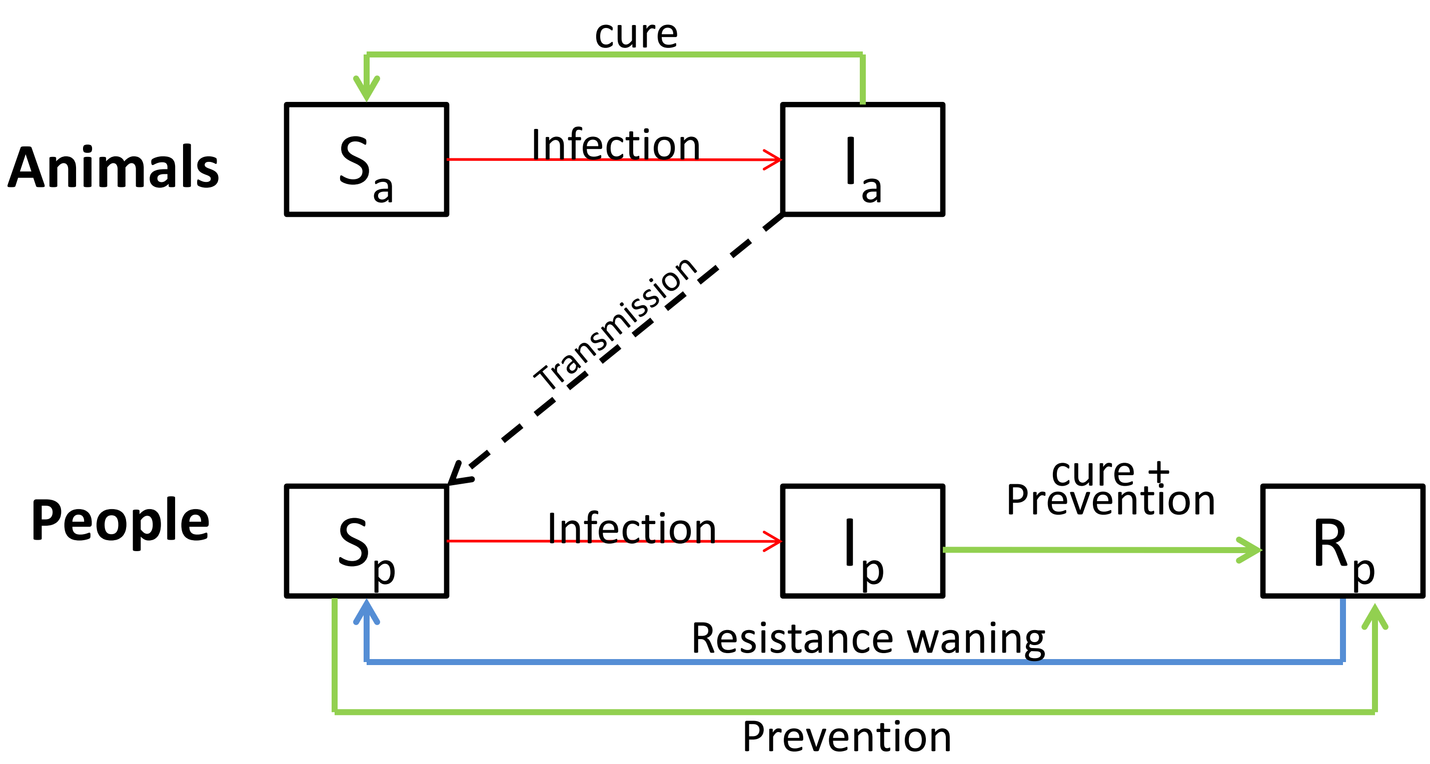
- Schistosomiasis is a parasitic disease caused by several species of trematodes
- Some trematodes can survive within human body as long as 40 years and harm human's health
- More than 800 thousand people are infected per year in China
- Snail-mediated transmission

Assumptions

- 1. The population of people and animals are both kept stable, which indicates that the natural birth and death rate are equal;
- 2. Due to low mortality, the disease-induced death rate for both animals and people is negligible;
- 3. The infected people (I_p) and animals (I_a) cannot recover without treatment;
- 4. Infection occurs only from animals to animals and from animals to people, but not among people or from people to animal

- 5. The infected animals (I_a) become susceptible (S_a) again after they are cured, without any resistance to the disease.
- 6. Infected people (I_p) always take prevention medicine after they are cured which give temporary resistance to schistosomiasis, while the susceptibles (S_p) can also receive prevention drug from public health service agencies and became removed people (R_p).
- 7. The resistance given by prevention drug will wane and disappear after a period

Transfer Diagram of Model



S_a: suseptible animals ; I_a: Infectious animals;
S_p: suseptible people; I_p: Infected people; R_p: removed people

ODEs

For animals:

Infection — Cure

$$dS_a(t)/dt = \boxed{- \beta_{aa} S_a(t) I_a(t)/N_a} + \boxed{\mu_a I_a(t)}$$

$$dI_a(t)/dt = \beta_{aa} S_a(t) I_a(t) /N_a - \mu_a I_a(t)$$

$$S_a(t) + I_a(t) = N_a$$

Resistance waning

For people:

Infection — Prevention

$$dS_p(t)/dt = \boxed{- \beta_{ap} S_p(t) I_a(t)/N_a} + \boxed{\omega_p R_p(t)} - p_p S_p(t)$$

$$dI_p(t)/dt = \beta_{ap} S_p(t) I_a(t) /N_a - \mu_p I_p(t)$$

$$dR_p(t)/dt = \mu_p I_p(t) + p_p S_p(t) - \omega_p R_p(t)$$

$$S_p(t) + I_p(t) + R_p(t) = N_p$$

cure + Prevention

Parameters

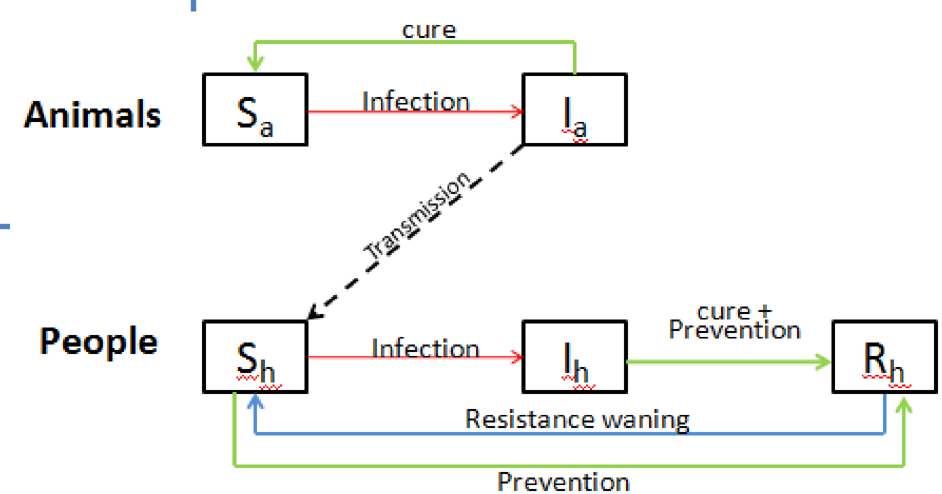
β_{aa} Transmission Coefficient among animals

β_{ap} Transmission Coefficient from animals topeople

μ_a, μ_p are the cure rates for animals and people

P_p is the prevention rate of people

W_p is the resistance waning rate of people



Non-dimensionalization

Let $s_a(t) = S_a(t)/N_a$; $i_a(t) = I_a(t)/N_a$

$s_p(t) = S_p(t)/Np$; $i_p(t) = I_p(t)/N_p$; $r_p(t) = R_p(t)/N_p$

$$ds_a(t)/dt = - \beta_{aa}\, s_a(t)\, i_a(t) + \mu_a\, i_a(t)$$

$$di_a(t)/dt = \beta_{aa}\, s_a(t)\, i_a(t) - \mu_a i_a(t)$$

$$s_a(t) + i_a(t) = 1$$

$$ds_p(t)/dt = - \beta_{ap}\, s_p(t)\, i_a(t) + \omega_p\, r_p(t) - p_p s_p(t)$$

$$di_p(t)/dt = \beta_{ap}\, s_p(t)\, i_a(t) - \mu_p\, i_p(t)$$

$$dr_p(t)/dt = \mu_p\, i_p(t) + p_p s_p(t) - \omega_p\, r_p(t)$$

$$s_p(t) + i_p(t) + r_p(t) = 1$$

Further simplification

- Using $r_p(t)= 1- s_p(t) - i_p(t)$; $i_a(t)= 1- s_a(t))$

- The ODEs can be simplified as

$$ds_p(t)/dt = - \beta_{ap}\, s_p(t)\, (1- s_a(t)) + \omega_p\, (1- s_p(t) - i_p(t))– p_p s_p(t)$$

$$di_p(t)/dt = \beta_{ap}\, s_p(t)\, (1- s_a(t)) - \mu_p\, i_p(t)$$

$$ds_a(t)/dt = - \beta_{aa}\, s_a(t)(1- s_a(t)) + \mu_a\,)(1- s_a(t))$$

Equilibrium Analysis

$0 = -\beta_{ap} s^*_p (1 - s^*_a) + \omega_p (1 - s^*_p - i^*_p) - p_p s^*_p$ (1)

$0 = \beta_{ap} s^*_p (1 - s^*_a) - \mu_p i^*_p$ (2)

$0 = -\beta_{aa} s^*_a (1 - s^*_a) + \mu_a (1 - s^*_a)$ (3)

1. $s_a^* = 1$	2. $s_a^* = \frac{\mu_a}{\beta_{aa}}$
Disease-free Equilibrium(DFE)	**Non Disease Free Equilibrium**
$s_p^* = \frac{\omega_p}{\omega_p + P_p}$, $i_p^* = 0$	$s_p^* = \frac{\omega_p \mu_p}{(\omega_p + P_p)\mu_p + (\omega_p + \mu_p)\beta_{ap}(1 - s_a^*)}$, $i_p^* = \frac{\omega_p \beta_{ap}(1 - s_a^*)}{(\omega_p + P_p)\mu_p + (\omega_p + \mu_p)\beta_{ap}(1 - s_a^*)}$

1. Disease-free Equilibrium Analysis

- Jacobian matrix

$$J(s_p^*, i_p^*, s_a^*) = \begin{pmatrix} -\omega_p - \beta_{ap}(1-s_a^*) - P_p & -\omega_p & \beta_{ap} s_p^* \\ \beta_{ap}(1-s_a^*) & -\mu_p & -\beta_{ap} s_p^* \\ 0 & 0 & -\mu_a - \beta_{aa} + 2\beta_{aa} s_a^* \end{pmatrix}$$

$$= \begin{pmatrix} -\omega_p - P_p & -\omega_p & \beta_{ap} \dfrac{\omega_p}{\omega_p + P_p} \\ 0 & -\mu_p & -\beta_{ap} \dfrac{\omega_p}{\omega_p + P_p} \\ 0 & 0 & -\mu_a + \beta_{aa} \end{pmatrix}$$

Disease-free Equilibrium Analysis

- Find the eigenvalues

$$\lambda_1 = -\omega_p - P_p < 0$$

$$\lambda_2 = -\mu_p < 0$$

$$\lambda_3 = -\mu_a + \beta_{aa} < 0, if \mu_a > \beta_{aa}$$

This fixed point will be stable if

$$\mu_a > \beta_{aa} \Rightarrow R_o = \frac{\beta_{aa}}{\mu_a} < 1$$

R_o

$$R_o = \frac{\beta_{aa}}{\mu_a}$$

β_{aa} Transmission Coefficient among animals
μ_a is the cure rate for animals

Physical meanings of Ro:

Decide the stability points of susceptible and infective human number and is the important factor for disease control.

2.Non Disease Free Equilibrium Analysis

- Jacobian matrix

$$J(s_p^*, i_p^*, s_a^*) = \begin{pmatrix} -\omega_p - \beta_{ap}(1-\frac{1}{R_0}) - P_p & -\omega_p & \dfrac{\omega_p \beta_{ap} \mu_p}{(\omega_p + P_p)\mu_p + (\omega_p + \mu_p)\beta_{ap}(1-\frac{1}{R_0})} \\ \beta_{ap}(1-\frac{1}{R_0}) & -\mu_p & \dfrac{\omega_p \beta_{ap} \mu_p}{(\omega_p + P_p)\mu_p + (\omega_p + \mu_p)\beta_{ap}(1-\frac{1}{R_0})} \\ 0 & 0 & -\mu_a - \beta_{aa} + \dfrac{2\beta_{aa}}{R_o} \end{pmatrix}$$

Non Disease Free Equilibrium Analysis

- Find the eigenvalues

$$\lambda_1 = -\mu_a - \beta_{aa} + \frac{2\beta_{aa}}{R_o} < 0, if R_0 > 1$$

$$\lambda_2 and \lambda_3 < 0, if R_0 > 1$$

Ro<=1, disease free equilibrium

Infective equals zero

Ro>1, non disease free equilibrium

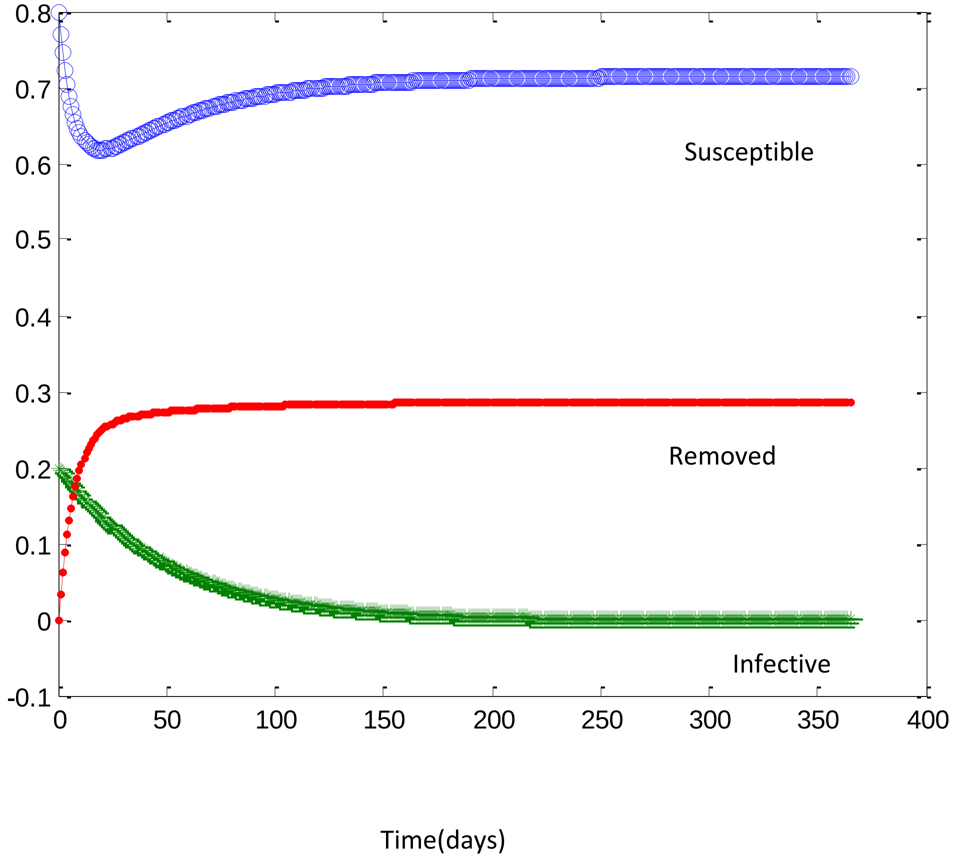
Infective is higher with Ro increasing.

Case 1 (Disease-free Equilibrium) (Ro<=1)

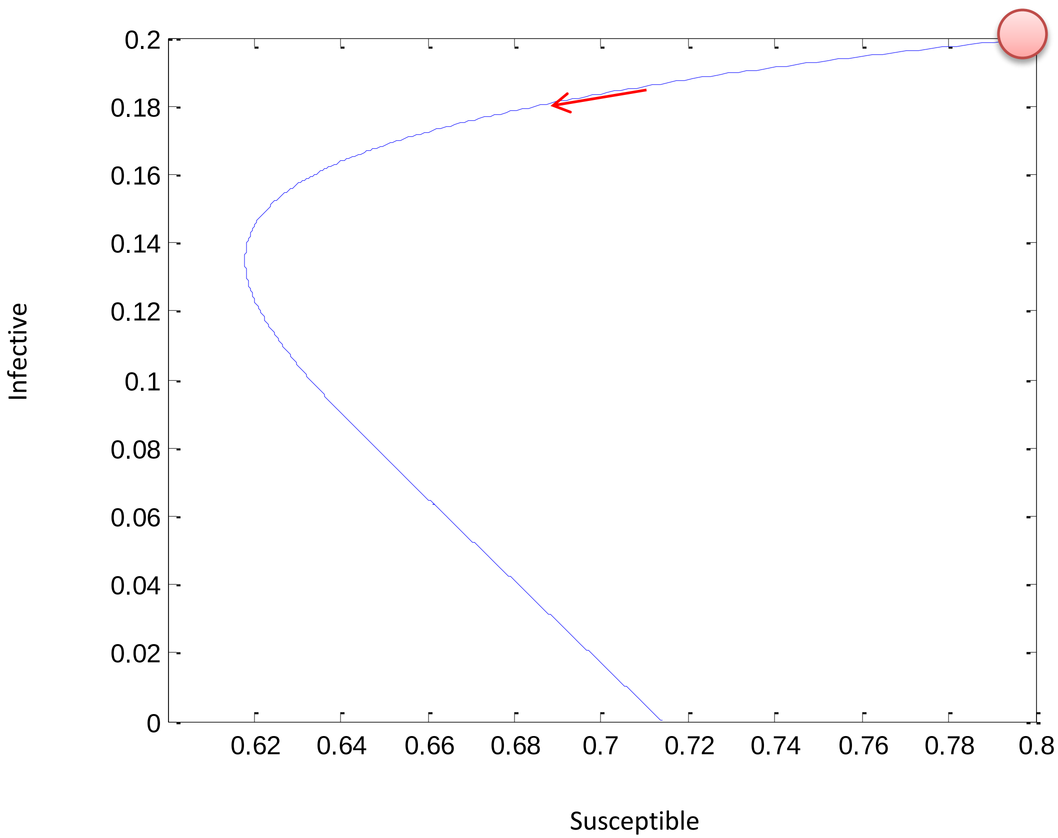
ω_p	0.1 day-1
β_{ap}	1.06 day-1
β_{aa}	1day-1
P_p	0.04 day-1
μ_p	0.02day-1
μ_a	10day-1
R_0	β_{aa} / μ_a **=1/10<1**

- After the stability of animals epidemics, human start the dispersion
- Stability point: **s*p=0.71, i*p=0**

For human
Initial (0.8,0.2,0)



For human
Initial (0.8,0.2,0)

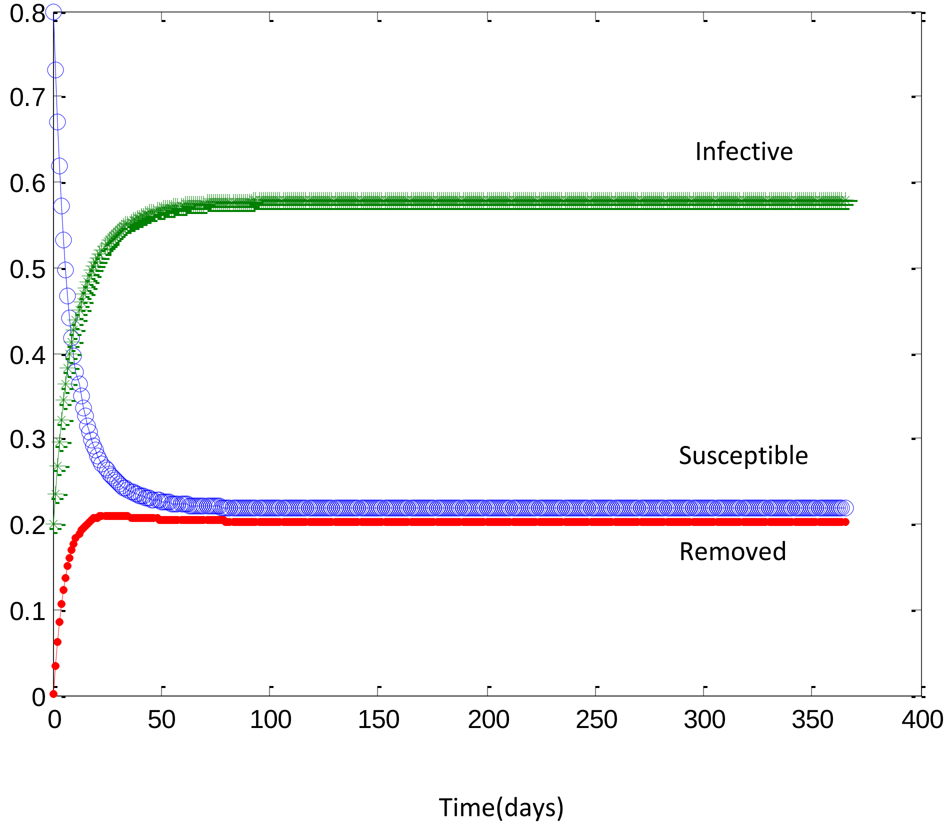


Case 2 (Disease-free Equilibrium)

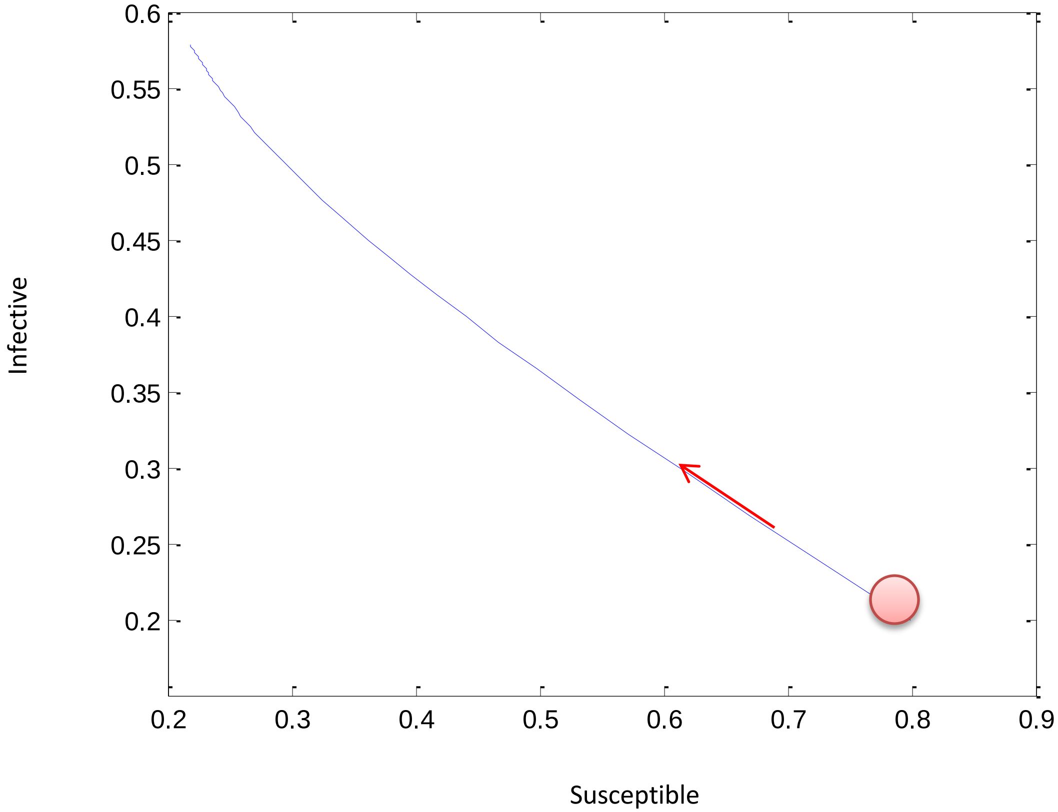
(Ro>=1)

ω_p	0.1 day-1
β_{ap}	1.06 day-1
β_{aa}	10.5 day-1
P_p	0.04 day-1
μ_p	0.02day-1
μ_a	10day-1
R_0	β_{aa} / μ_a =**10.5/10=1.05>1**
•After the stability of animals epidemics, human start the dispersion •Stability point: **s*p=0.23, i*p=0.57**	

For human
Initial (0.8,0.2,0)



For human
Initial (0.8,0.2,0)

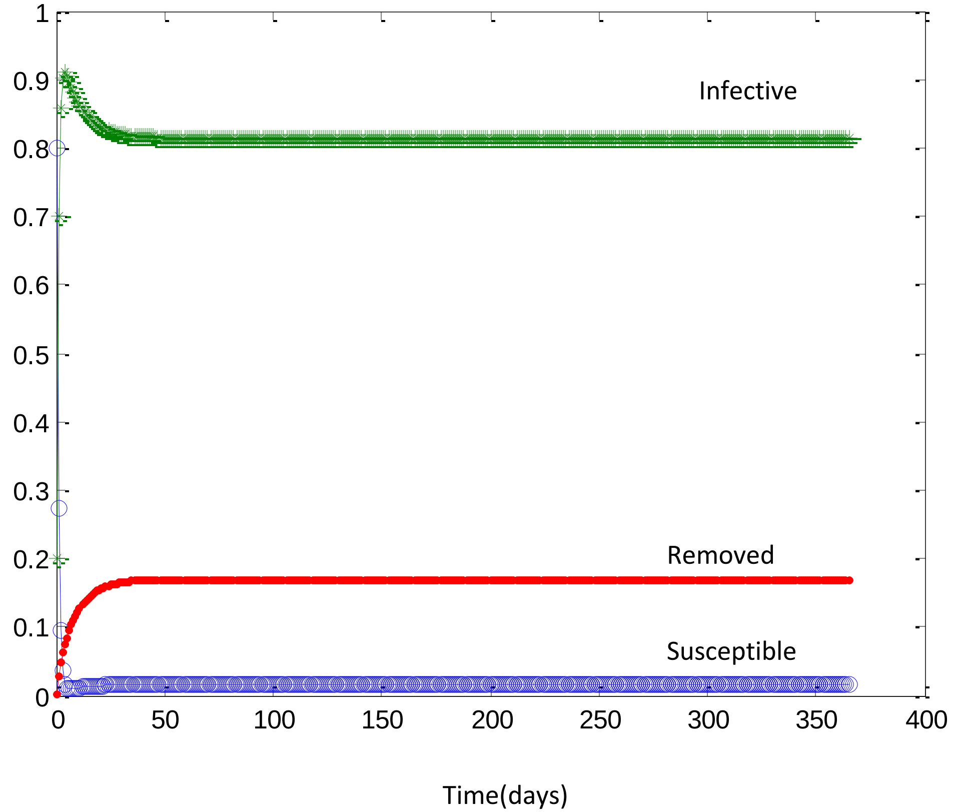


Case 3 (Disease-free Equilibrium)

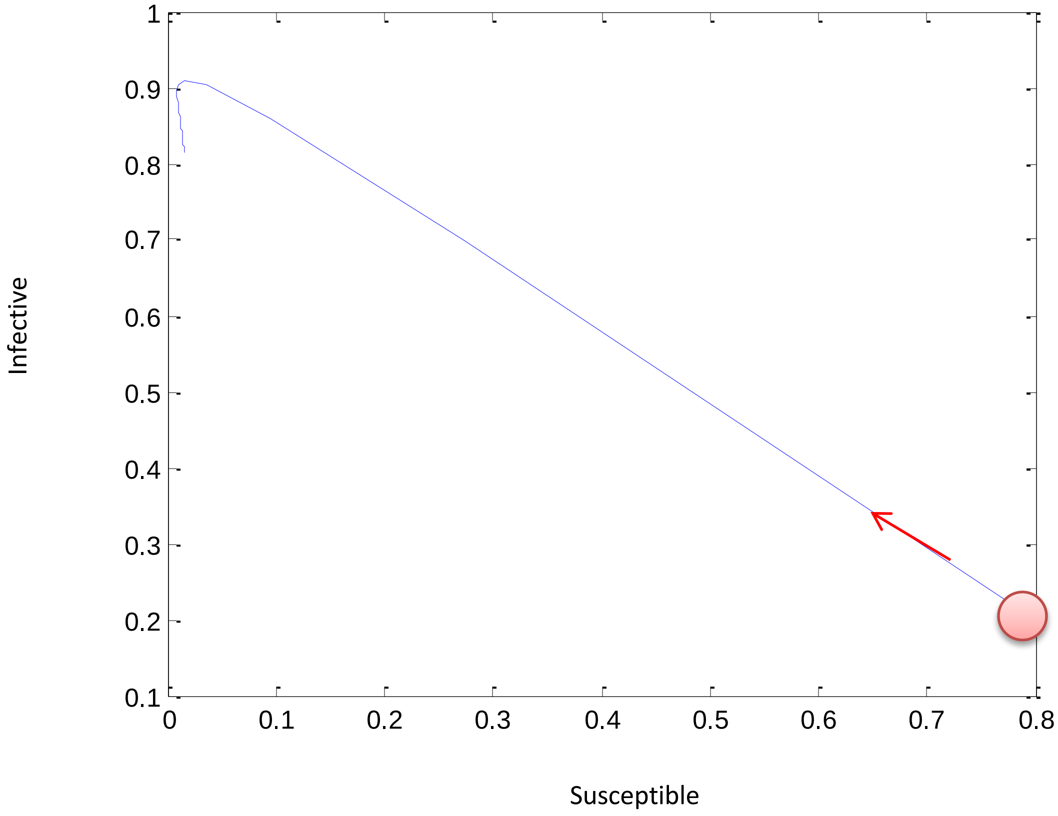
(Ro>=1)

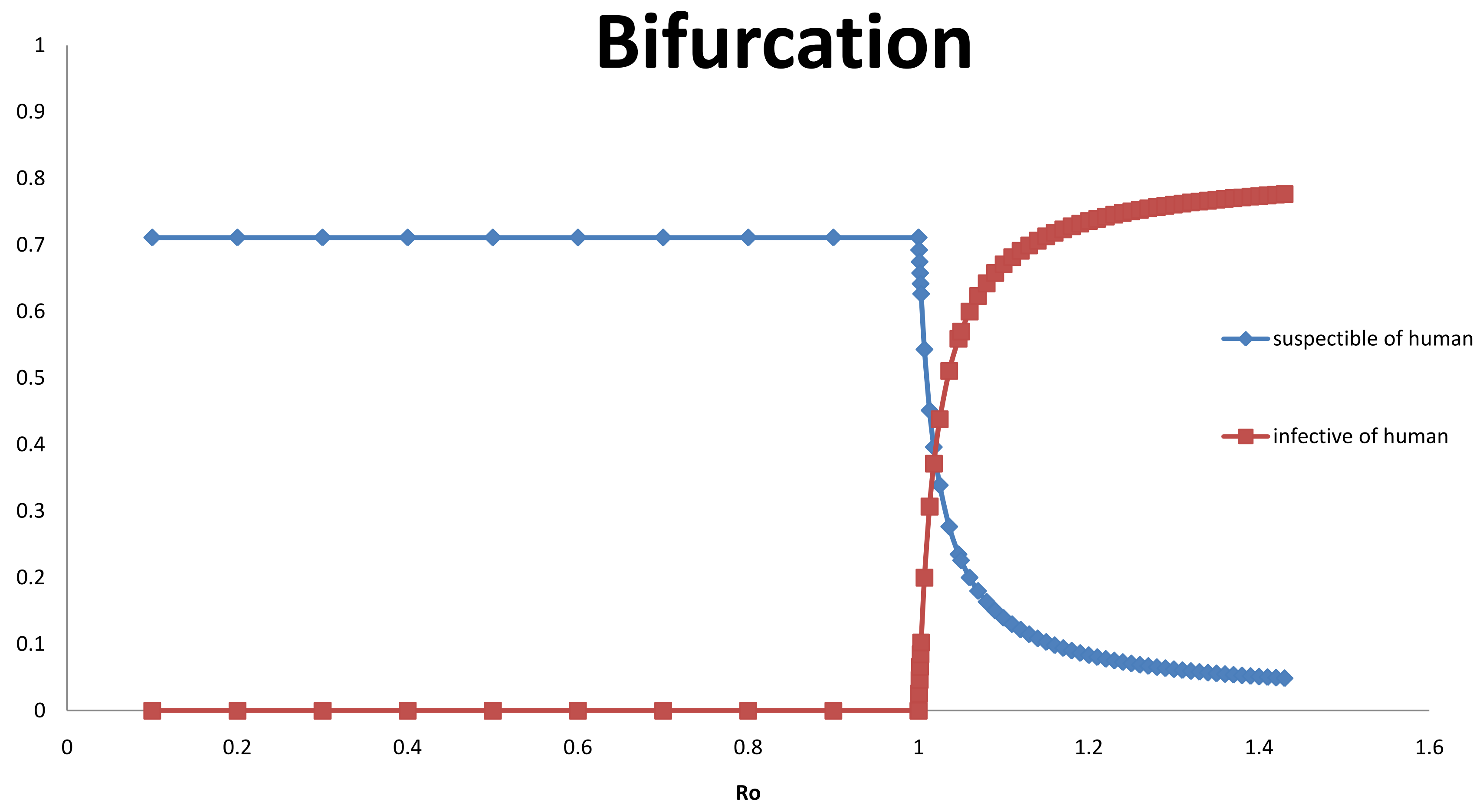
ω_p	0.1 day-1
β_{ap}	1.06 day-1
β_{aa}	500 day-1
P_p	0.04 day-1
μ_p	0.02day-1
μ_a	10day-1
R_0	β_{aa} / μ_a =**500/10=50>1**
•After the stability of animals epidemics, human start the dispersion •Stability point: **s*p=0.02, i*p=0.82**	

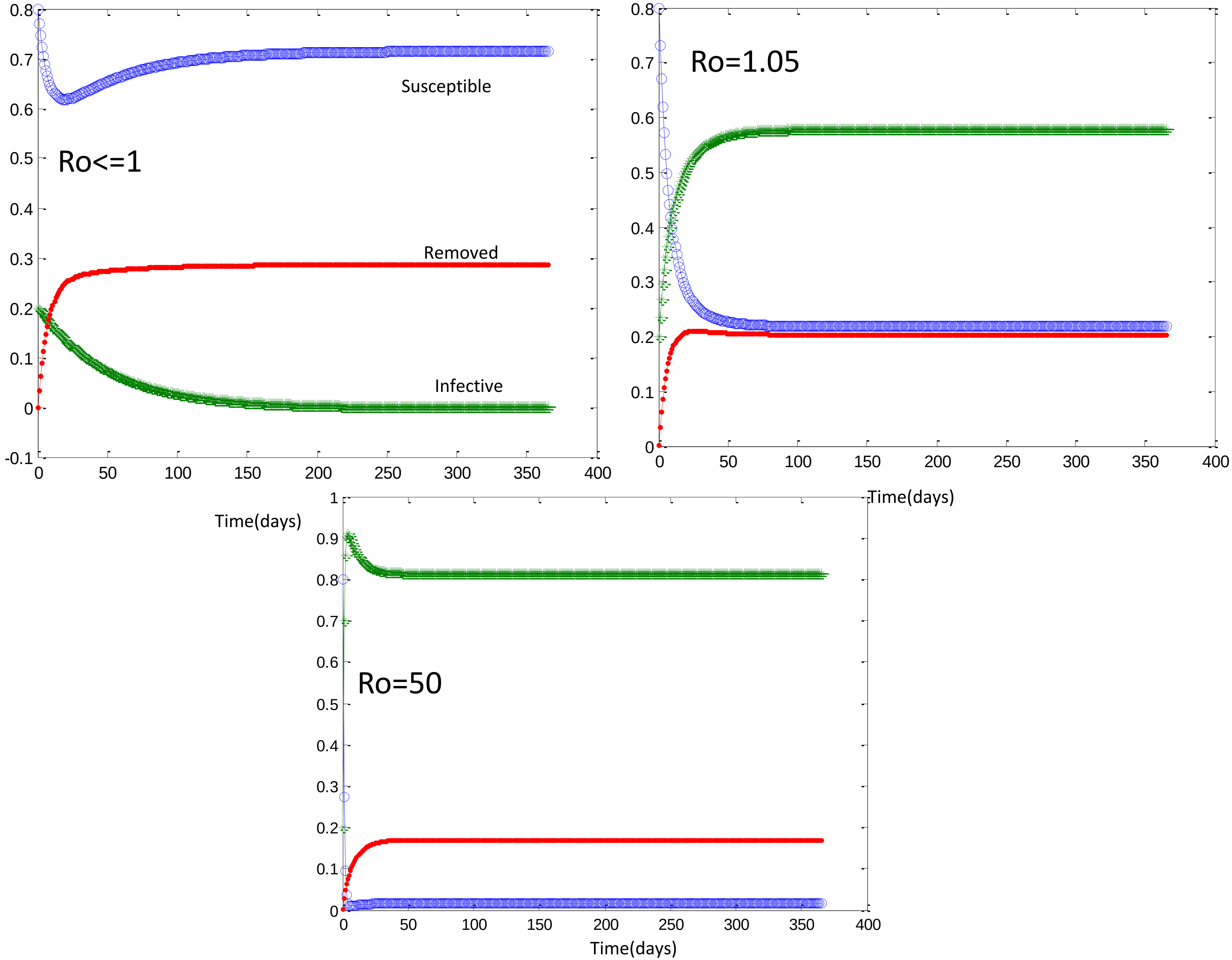
For human
Initial (0.8,0.2,0)



For human
Initial (0.8,0.2,0)




Bifurcation
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Ro
suspectible of human
infective of human



Susceptible
Ro<=1
Removed
Infective
Ro=1.05
Time(days)
Time(days)
Ro=50
Time(days)

Conclusions

1. There are two fixed points.
2. When R_o<=1, the disease-free equilibrium point is stable and the disease will disappear after certain time; when R_o>1, the Non disease-free equilibrium point is stable and the disease will disperse and stabilize in a local area.
3. Suggestion: control the R_o value under 1 to decrease the dispersion of schistosomiasis disease by two ways, which are to increase the recovery rate of animals and reduce the transmission rate of disease among animals by hunting or isolating epidemic media, snails.

Reference

- En.wikipedia.org/wiki/Schistosomiasis.
- Liu, H., Z. Li, H. Liu, L. Huang, and W. Wang. 2008. Mathematical model of rabies and its controlling strategies in China. Journal of Lanzhou University (Natural Sciences), (44):4.
- Murray, J.D. 2002. Mathematical_Biology I: An Introduction. New York: Springer-Verlag,
- Vries, G., T. Hillen, M. Lewis, and B. Schonfisch. 2006. A Course in Mathematical Biology: Quantitative Modeling with Mathematical and Computational. SIAM.
- Wiraningsih, E.D., Widodo, L. Aryati, S. Toaha, and S. Lenhart. 2010. Optimal Control for SEIR Rabies Model between Dogs and Human with Vaccination Effect in Dogs. Proceedings of the 6th IMT-GT Conference on Mathematics, Statistics and its Application (ICMSA2010).
- Zhao, Y. 2009. Modeling avian influenza in bird-human system. Master's thesis. Massey University, New Zealand.